Animating Virtual Characters

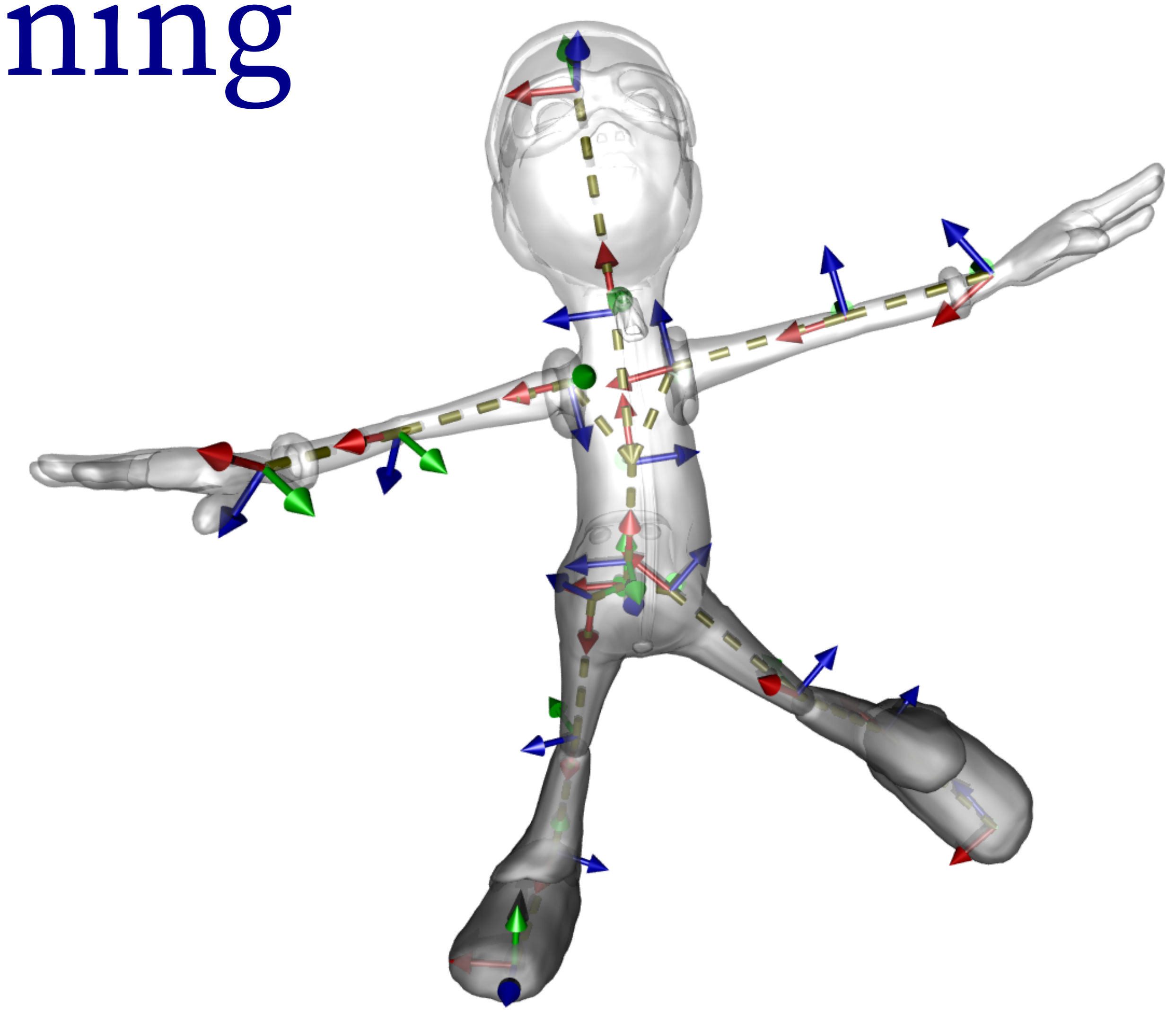
Skin deformation

Skeleton based deformation - Skinning

Objective: Deform articulated character

Idea: Use skeleton to control limbs

Articulations as rotations



Animation Skeleton

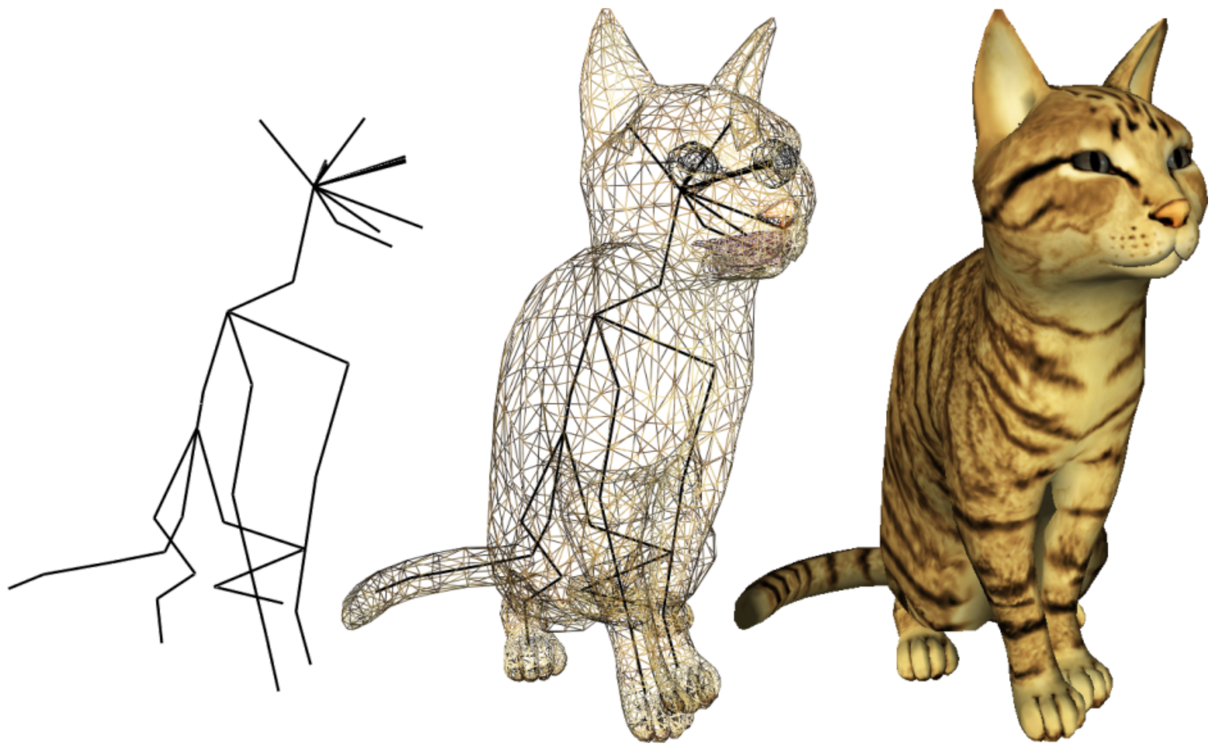
Set of frames T_i: position, orientation

Describe non-rigid parts of the character (dof)

Terminology

Joint = A frame T_i

Bone = Segment between two joints

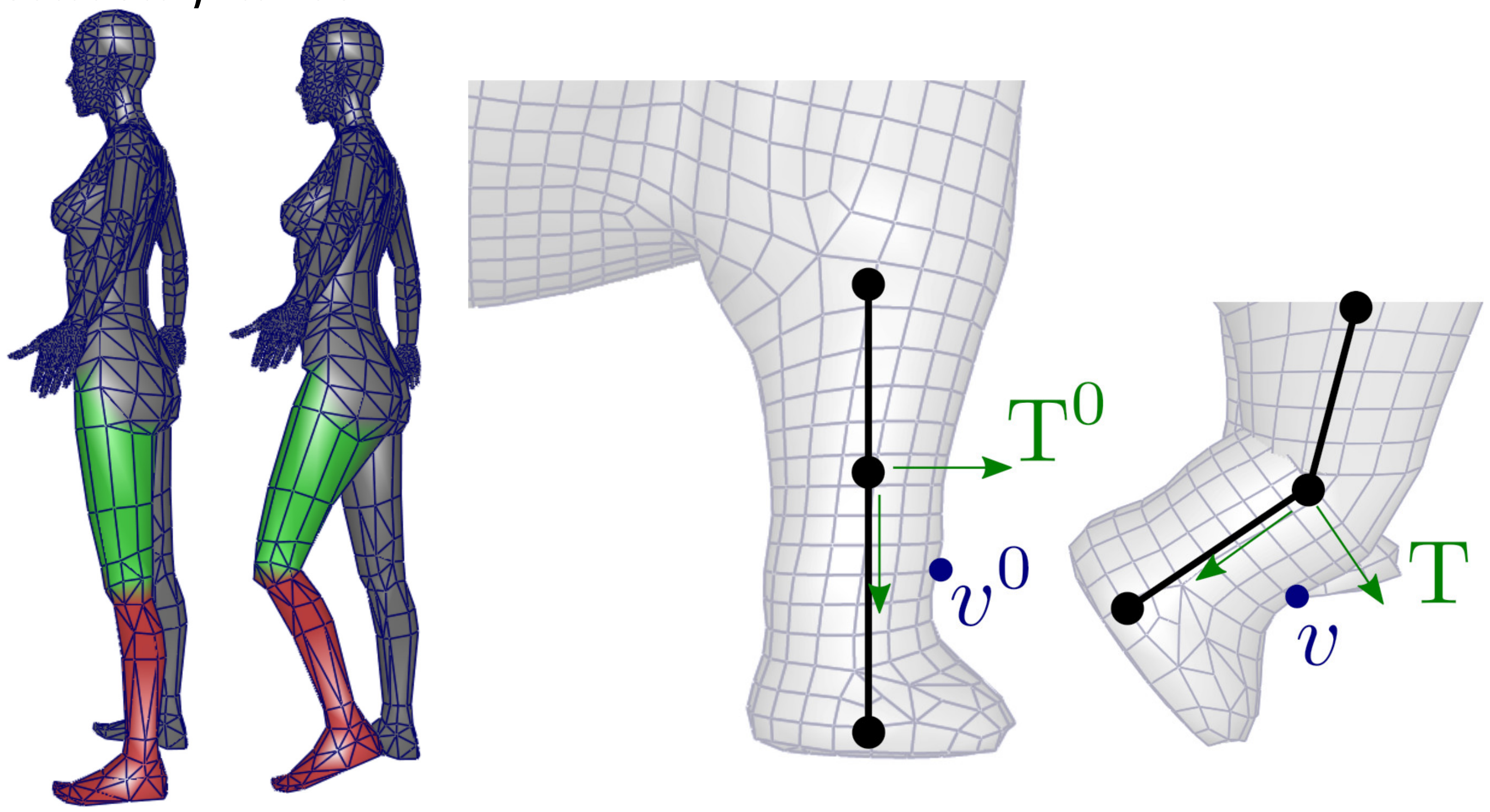


Note: Animation skeleton $\neq$ Anatomical skeleton

Rigid skinning

Attach rigidly subset of vertices to specific bones, described by its root joint/frame.

Vertices are following rigid deformation of their associated frame.



Deformation formulation

Consider, at rest pose/initial state

- A frame T^0
- A vertex v^0 attached to this frame

After deformation

- New frame T
- New vertex position v

Question: What are the new coordinates v w/r T, T^0, v^0 ?

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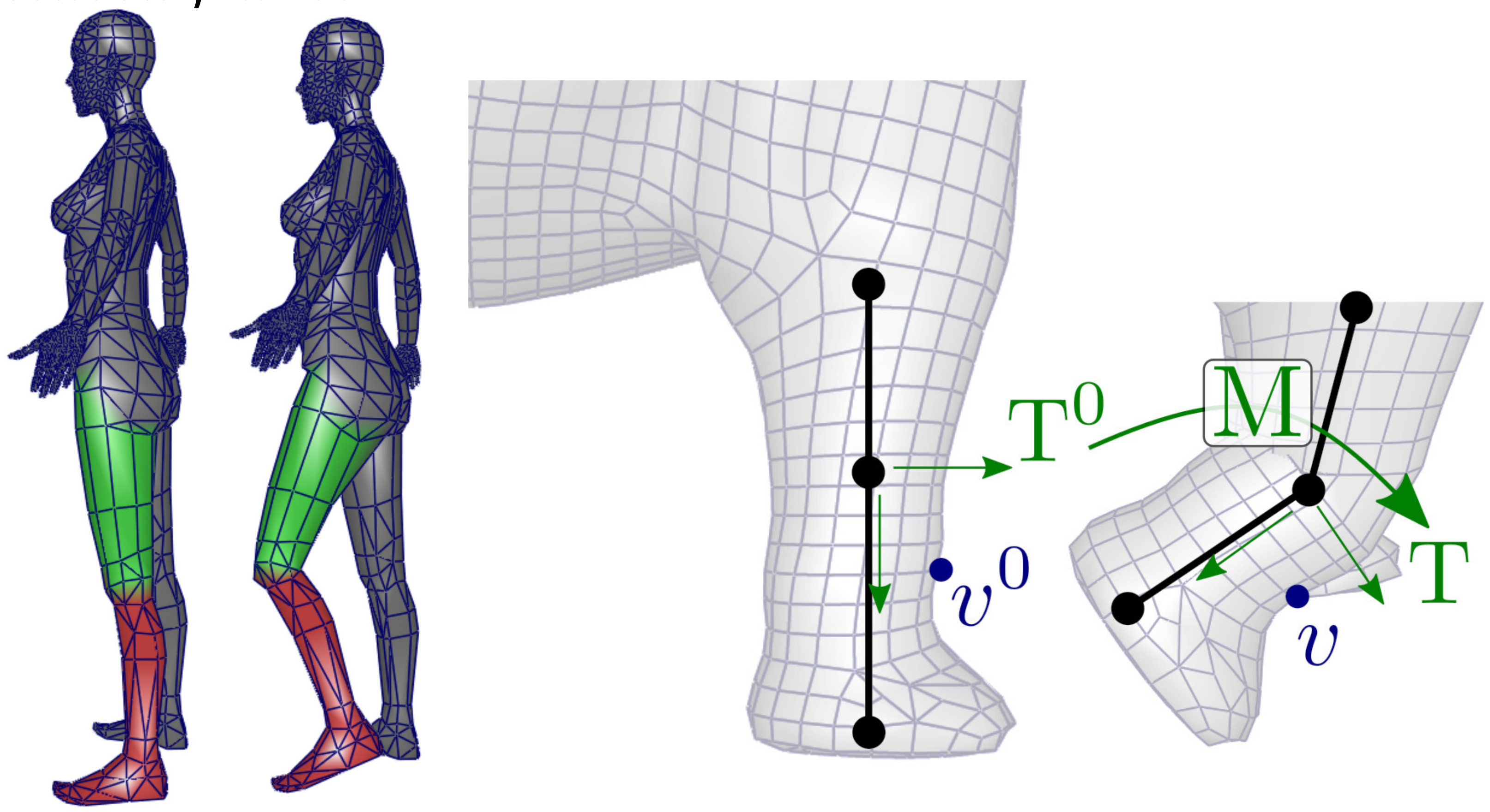


Question: What are the new coordinates v w/r T, T^0, v^0 ?

v^0 and v have similar local coordinates w/r to T^0 and T

$$\Rightarrow \mathrm{T}^{-1} v = (\mathrm{T}^0)^{-1} v_0$$

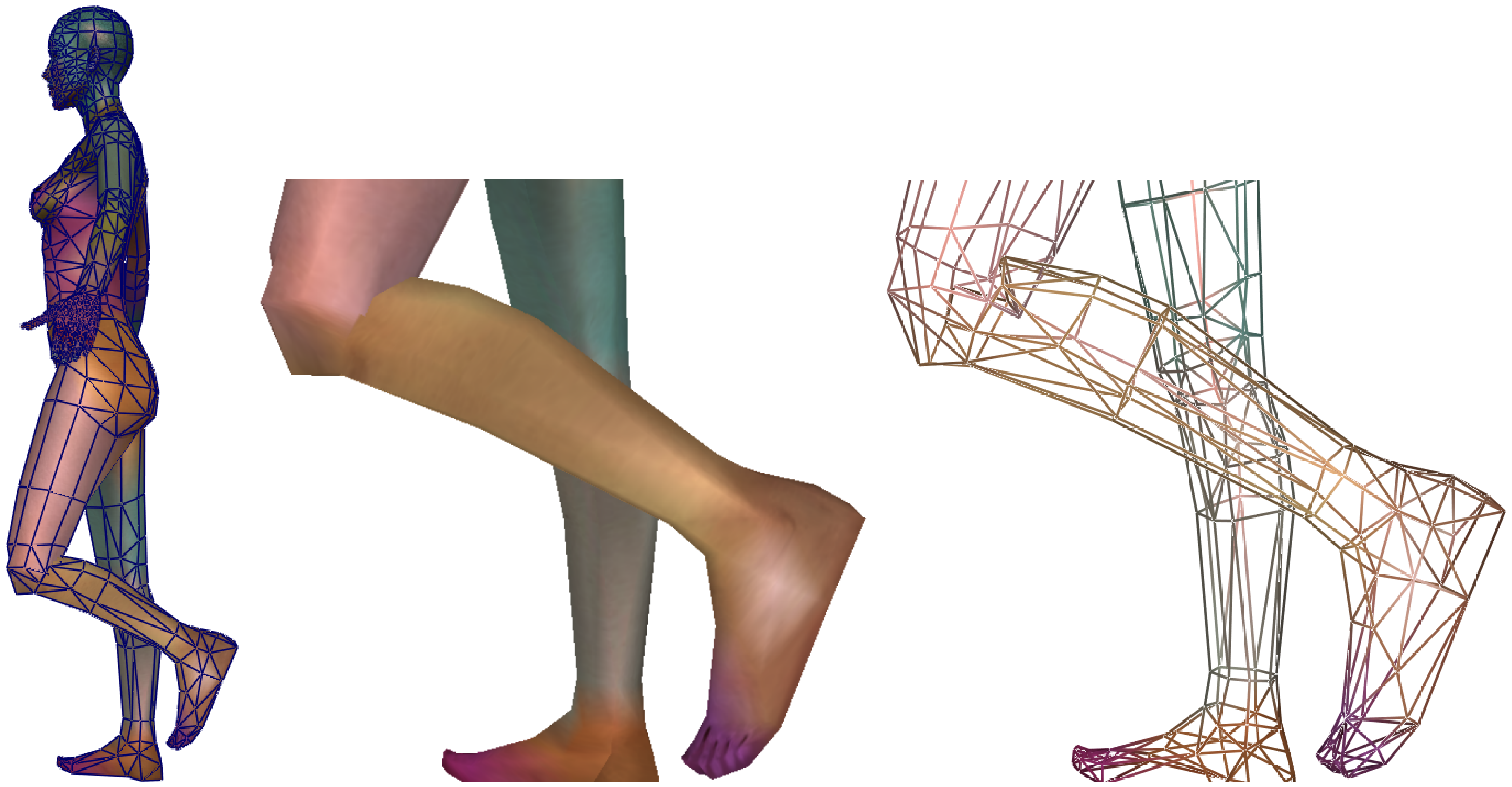
$$\boxed{\Rightarrow v = \mathrm{T}\,(\mathrm{T}^0)^{-1} v_0 = \mathrm{M}\, v_0}$$

Rigid Skinning

(+) Skeleton is easy to build

(+) Skeleton interaction is intuitive to model rigid articulation

(-) Discontinuities/Inter-penetrations



Idea of smooth skinning: Blend discontinuous transformation around articulation

Smooth skinning

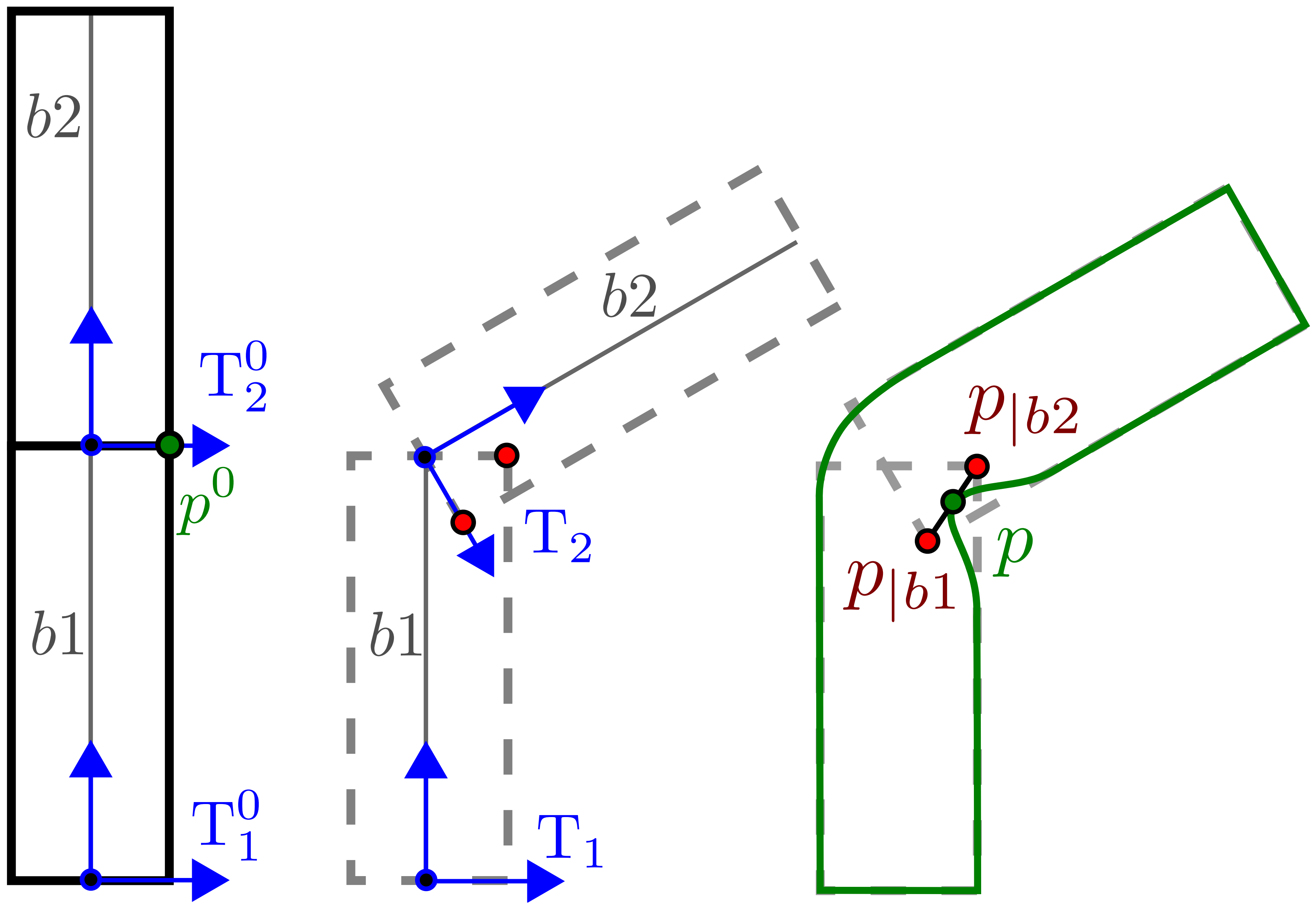
Linear Blend Skinning (LBS): Linear interpolation of positions between associated frames

Example at middle vertex position of a bending cylinder

$$p = 0.5\, p_{|b1} + 0.5\, p_{|b2}$$

$$p = 0.5\, \underbrace{\mathrm{T}_1\, (\mathrm{T}_1^0)^{-1}}_{\mathrm{M}_1}\, p^0 + 0.5\, \underbrace{\mathrm{T}_2\, (\mathrm{T}_2^0)^{-1}}_{\mathrm{M}_2}\, p^0$$

$$p = (0.5\, \mathrm{M}_1 + 0.5\, \mathrm{M}_2)\, p^0$$



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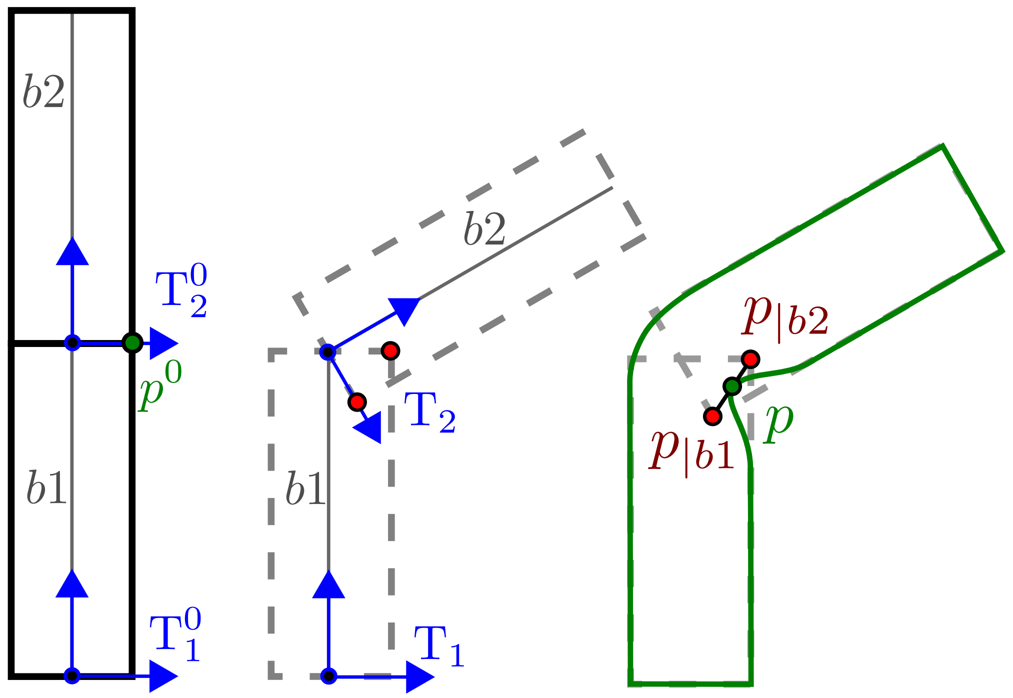
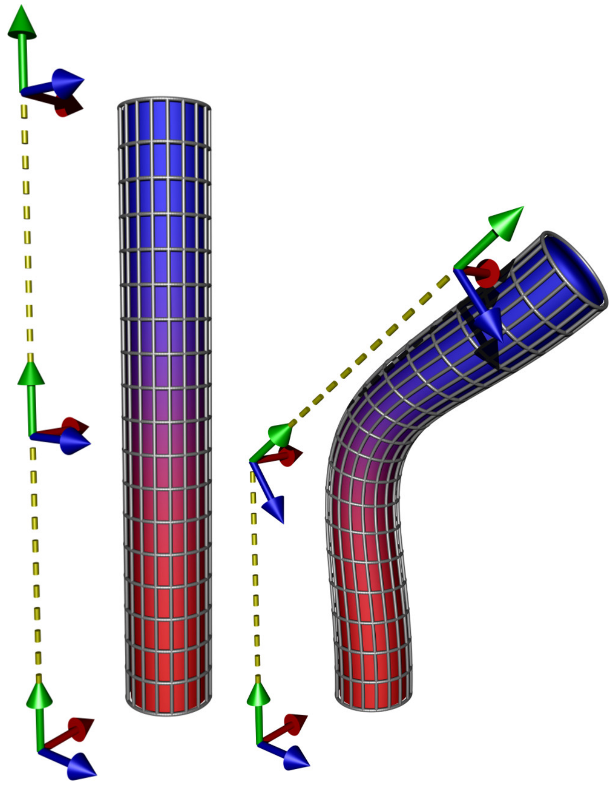


Can be generalized to arbitrary interpolation between two bones

$$p = \alpha\, p_{|b1} + (1-\alpha)\, p_{|b2} = (\alpha \mathrm{M}_1 + (1-\alpha)\, \mathrm{M}_2)\ p^0$$

α: skinning weights



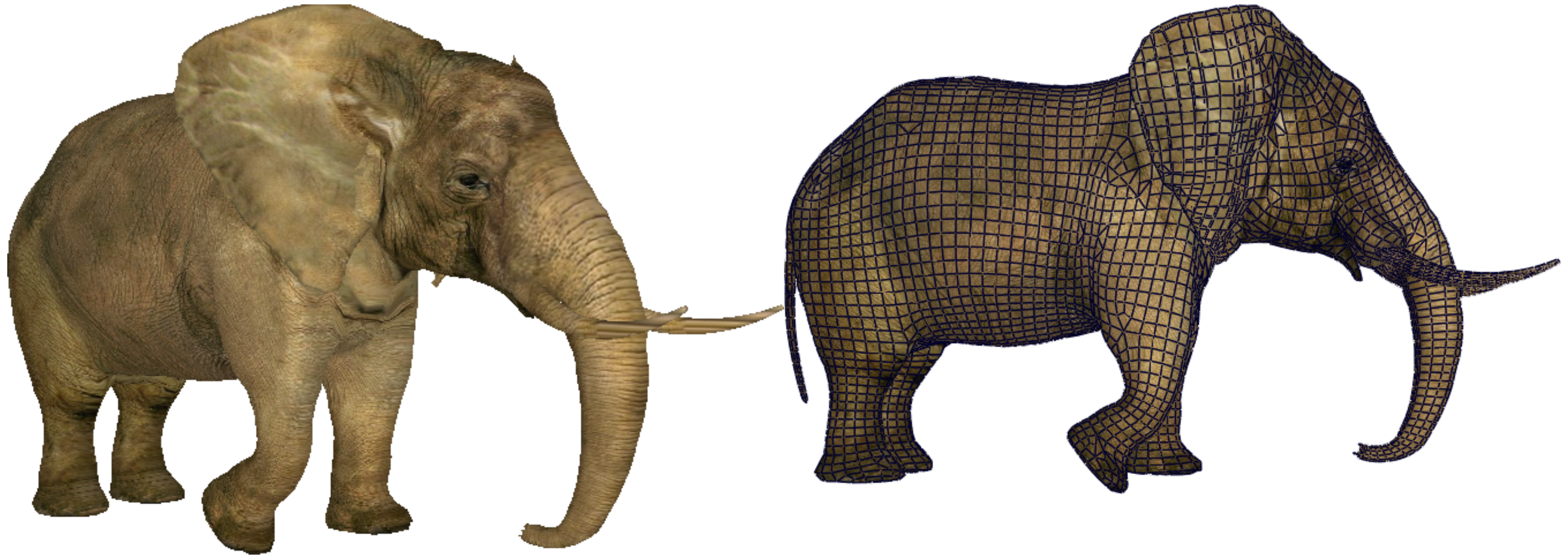
Can be generalized to any number of bones

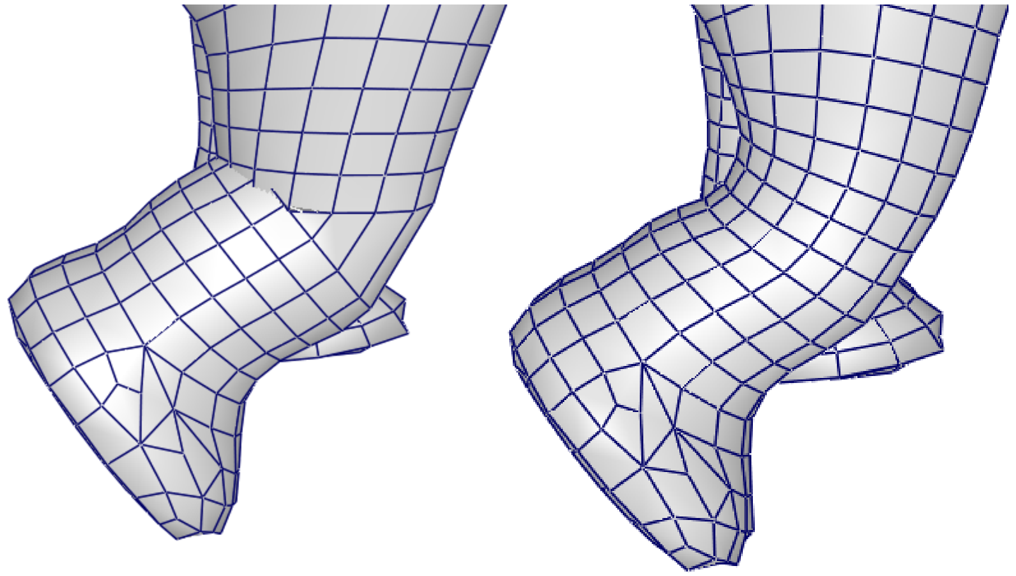
$$\boxed{p = \sum_{k=0}^{N-1} \alpha_k\ p_{|bk} = \left(\sum_{k=0}^{N-1} \alpha_k\, \mathrm{M}_k\right)\ p^0}\ ,\quad \sum_k \alpha_k = 1$$

Smooth skinning - Summary

$$p = \left(\sum_{k=0}^{N-1} \alpha_k \, \mathrm{M}_k\right) p^0 = \left(\sum_{k=0}^{N-1} \alpha_k \, \mathrm{T}_k \, (\mathrm{T}_k^0)^{-1}\right) p^0$$

$$\forall k \,,\ \omega_k \in [0,1], \text{ and } \sum_{k=0}^{N-1} \alpha_k = 1$$





The current **standard** for almost all articulated character deformations

- Intuitive deformation
- Controlable shape (*through weights*)
- Fast to compute (GPU compatible)
  matrix average, multiplication matrix-vector

⇒ Heavily used in Animation cinema & Video Game

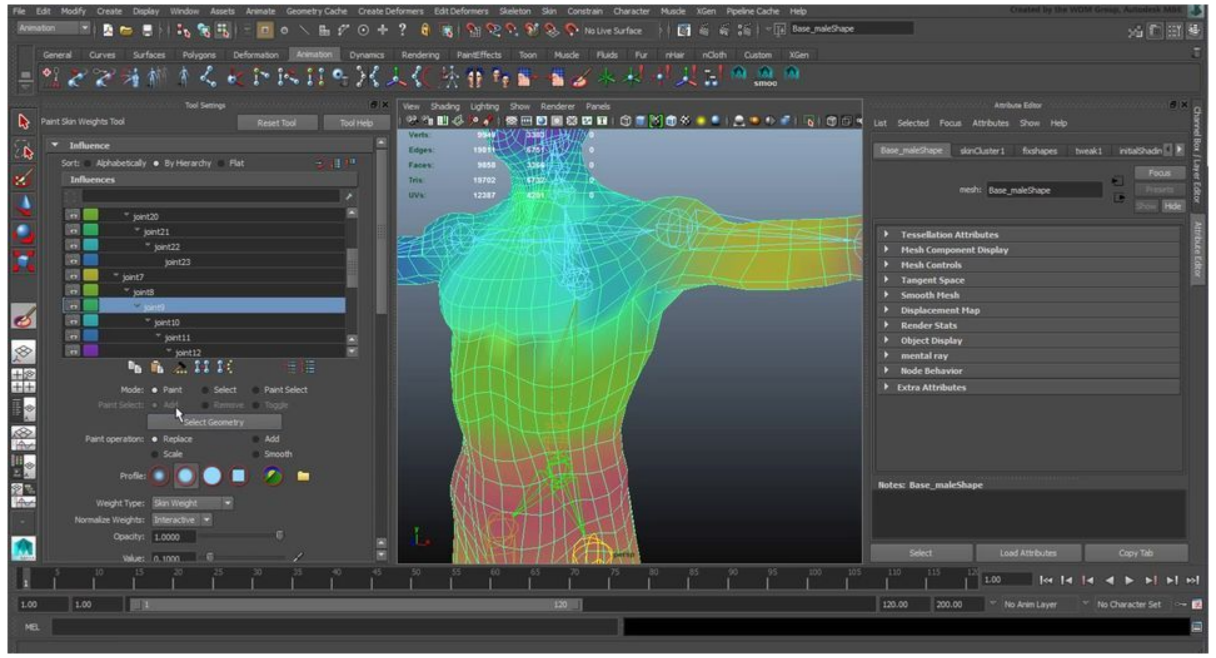
[Joint-Dependent Local Deformations for Hand Animation and Object Grasping. Nadia Magnenat-Thalmann, Rochard Laperière, Daniel Thalmann. Graphics Interface, 1988]

[Over My Dead, Polygonal Body. Jeff Lander. Game Developer Magazine, 1998]

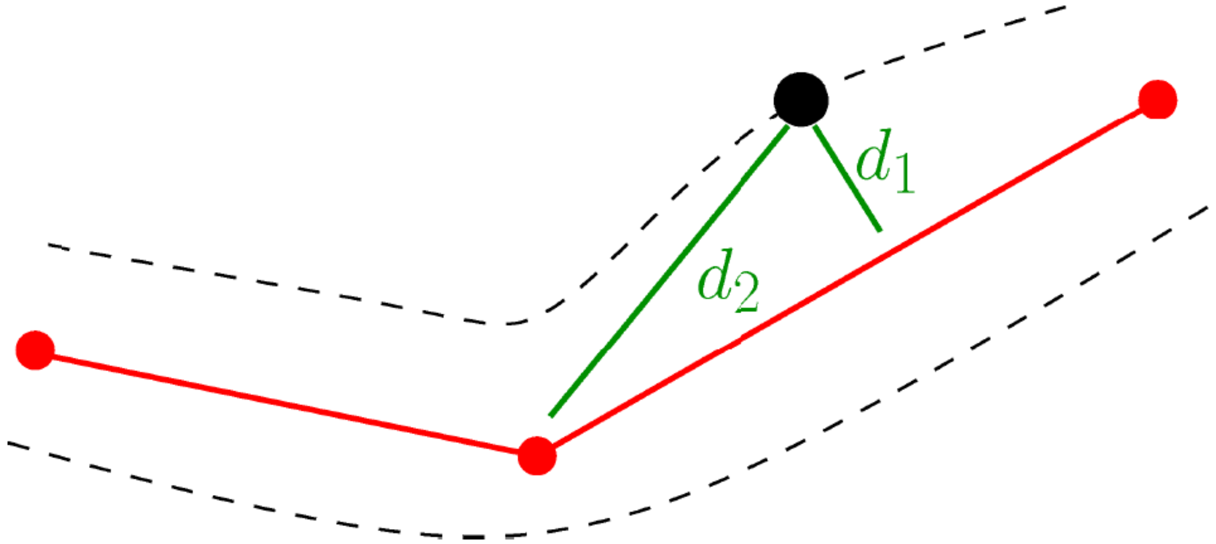
Skinning weights

How to generate skinning weights ?

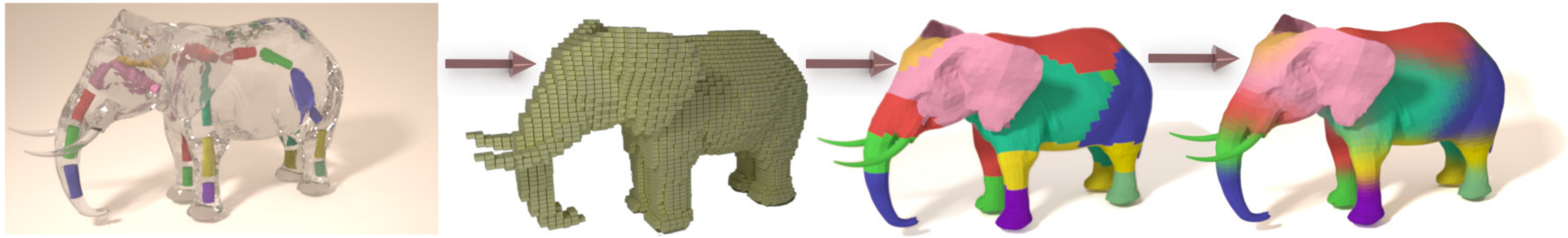
- Paint them manually
- Automatic computation



ex. Using cartesian distances: $\alpha_1 = d_1^{-1}/(d_1^{-1} + d_2^{-1})$



Or using diffusion on the volume/surface



Rigging: Associating bones and skinning weights (or any animation handle) to mesh parts

Skinning: File Format

Unfortunately few standard open format to store skinning animation data

- Main open format: Collada (XML), glTF (JSON)
- Common software related formats: FBX, Blend, 3DS, ...

krissie Maya 7.0 | ColladaMaya v2.04 | FCollada v1.14 Collada Maya Export Options:
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.json

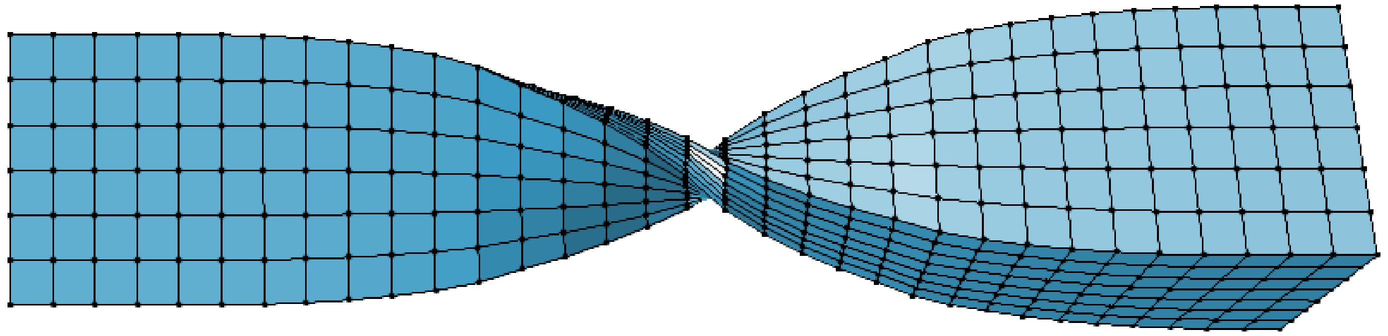
Linear Blend Skinning - Limitations

- Non-trivial rigging settings
- Artifacts for large rotations: Candy wrapper, Collapsing ellbow

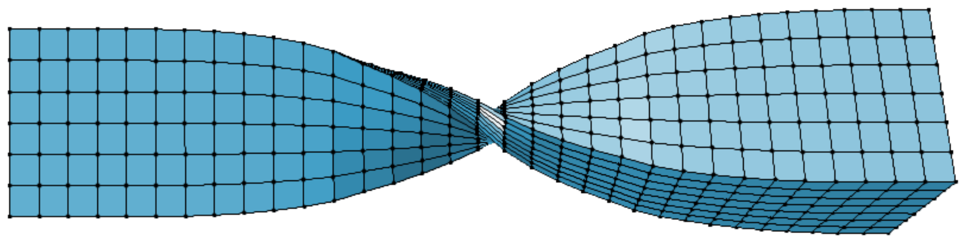
Candy wrapper

Collapsing elbow

Linear blending between rigid transformation matrices



Skinning improvement



Avoid linear blending b/w affine transform matrices

First idea: Split rigid transformation matrices in

- rotation part: blend using quaternions (solve candy wrapper)
- translation part: blend linearly

But doesn't work for general deformation: [redacted]

Rotation and translations are treated sep[redacted]

Possibility to compute an optimal center of rotation at each frame

[Spherical Blend Skinning: A Real-time Deformation of Articulated Models. L. Kavan, J. Zara. I3D 2005.]

Skinning improvement: Dual Quaternion

Second idea: Use of **dual quaternion**

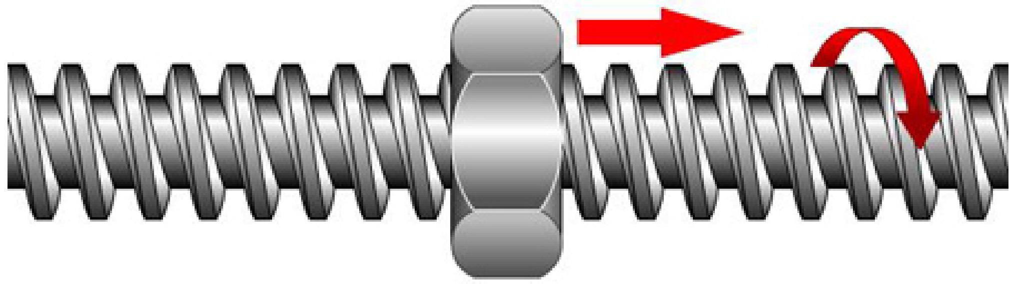
$\hat{q} = q_0 + \epsilon\, q_\epsilon$, ϵ dual element.

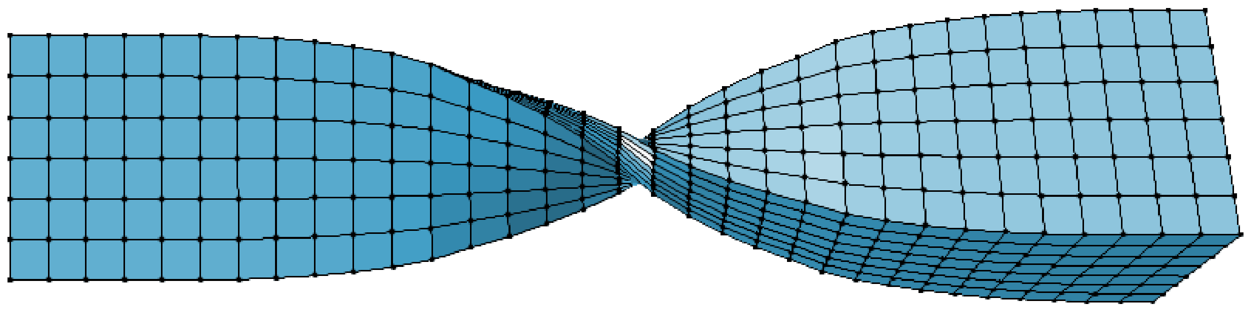
Dual quaternion = generalization of quaternion to handle *rigid transformation*

Rotation + translation

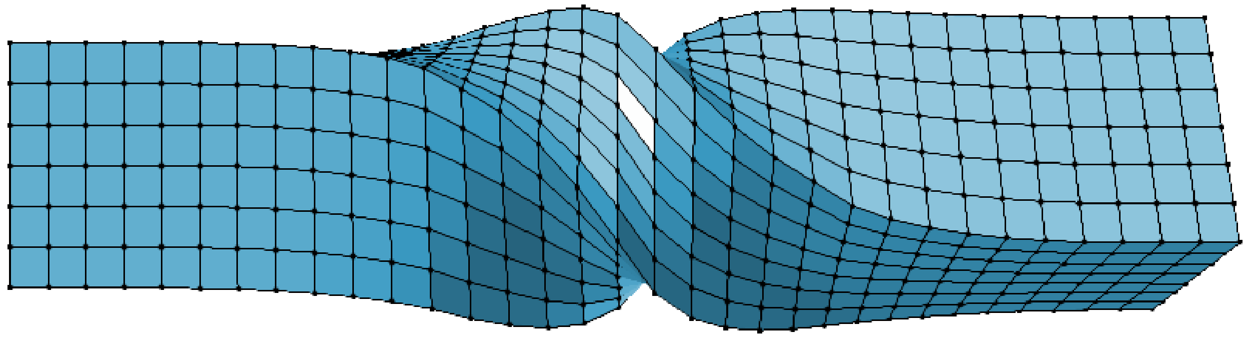
Unit dual quaternion models rigid transformation as **screw motions**

= *rotation about an axis followed by a translation in the direction of this axis*





$\Rightarrow$



LBS *DQS*

Dual number and dual quaternion

Dual number $a = a_0 + \epsilon a_\epsilon$

Dual element ϵ is nilpotent : $\epsilon \neq 0, \epsilon^2 = 0$

ϵ commonly used to model infinitesimal quantity (ex. automatic differentiation)

Dual quaternion $\hat{q}$ = Generalization of quaternion to dual numbers

$\hat{q} = q_0 + \epsilon\, q_\epsilon$

- q_0: pure rotation component
- q_ϵ: encodes translation component (dual part)

Given a unit quaternion q_0, and a translation $t = (t_x, t_y, t_z)$, the associated unit dual quaternion is

$$\boxed{\hat{q} = q_0 + \frac{\epsilon}{2}\, q_t\, q_0} \qquad q_t = (t_x, t_y, t_z, 0)$$

Unit dual quaternion ($\|\hat{q}\| = 1$) describe the set of rigid transformation as screw motion.

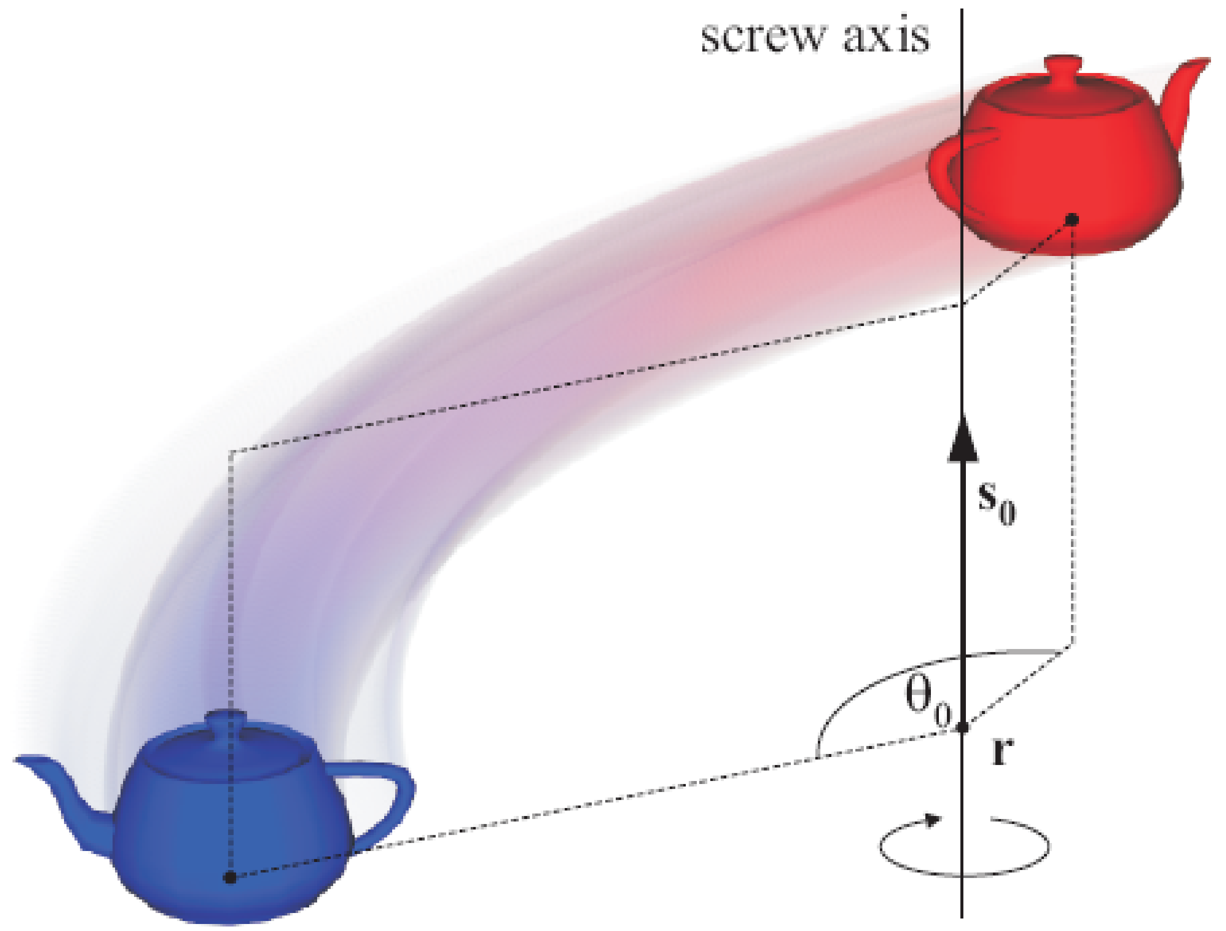
Axis-angle representation with dual quaternion

Similarily to quaternion: "angle/axis" correspondance

- $\hat{q} = \cos(\hat{\theta}/2) + \hat{n}\,\sin(\hat{\theta}/2)$

- $\hat{\theta} = \theta_0 + \epsilon\theta_\epsilon$
  - θ_0: angle of rotation
  - $\theta_\epsilon = t \cdot n_0$: quantity of translation along n_0

- $\hat{n} = n_0 + \epsilon\, n_\epsilon$
  - n_0: axis of rotation
  - $n_\epsilon = \frac{1}{2}((n_0 \times t)\,\mathrm{cotan}(\theta_0/2) + t) \times n_0$: called the moment of the rotation axis.



Convert dual quaterion to rotation/translation

Given a non-unit dual quaternion as input $\hat{q}' = q_0' + \epsilon q_\epsilon'$

How to compute the components of $\hat{q}'/\|\hat{q}'\| = \hat{q} = q_0 + \epsilon q_\epsilon$?

Express the parameterization in rotation-translation:

$$\hat{q} = q_0 + \frac{\epsilon}{2} q_t\, q_0 \ , \quad q_t = (t_x, t_y, t_z, 0)$$

First, normalize the non dual component: $\hat{q}'/\|q_0'\|$

$$\Rightarrow \boxed{q_0 = q_0'/\|q_0'\|}$$

Second, enforce the parameterization of the dual component: $\frac{1}{2} q_t\, q_0 = q_\epsilon'/\|q_0'\|$

$$\Rightarrow \boxed{q_t = 2\, q_\epsilon'\, q_0^\star/\|q_0'\|}$$

Dual Quaternion Skinning (DQS)

- Encode rigid transformation (q^i, t^i) into dual quaternion

$$\hat{q}^{\,i} = q_0^i + \frac{\epsilon}{2} q_t^i\, q_0^i \ , \quad q_t^i = (t_x^i, t_y^i, t_z^i, 0)$$

- Compute blending in the dual quaternion space (ScLERP)

$$\hat{q}^{\,\prime} = q_0^{\prime} + \epsilon\, q_\epsilon^{\prime} = \sum_i \omega_i\ \hat{q}^{\,i}$$

- Extract components (q, t) from $\hat{q}^{\,\prime}$

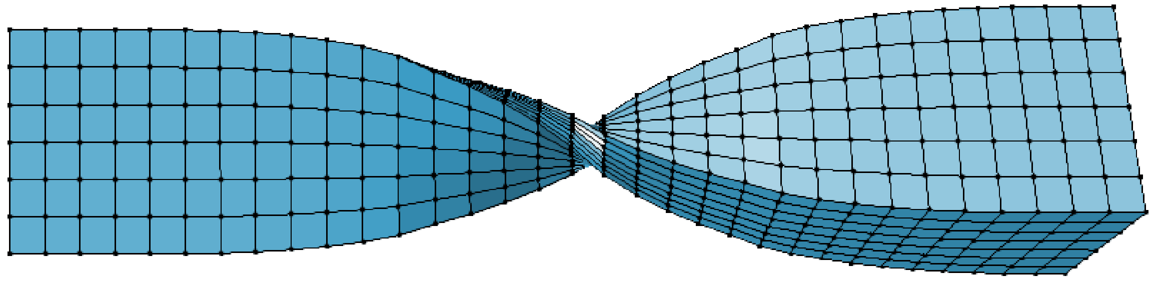
Compute $\hat{q}^{\,\prime\prime} = \hat{q}^{\,\prime} / \|q_0^{\prime}\| = q_0^{\prime\prime} + \epsilon q_\epsilon^{\prime\prime}$

$\Rightarrow q = q_0^{\prime\prime}$

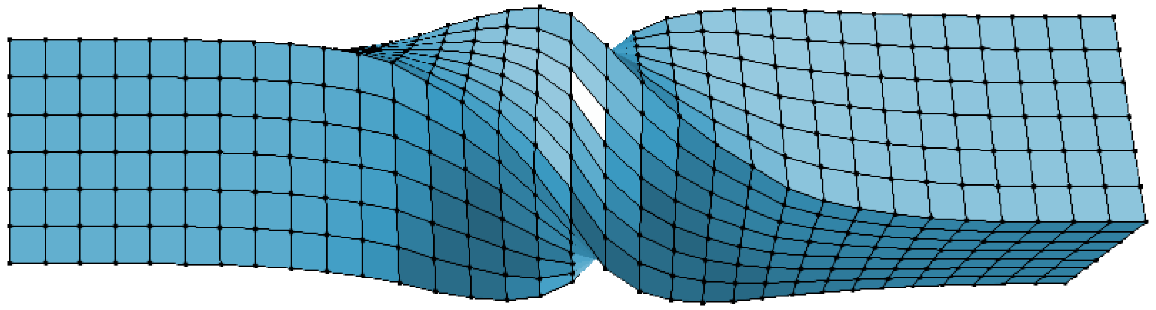
$\Rightarrow (t_x, t_y, t_z, 0) = 2\, q_\epsilon^{\prime\prime}\, q_0^{\star}$

- Apply finaly the transformation (q, t) to position p_0

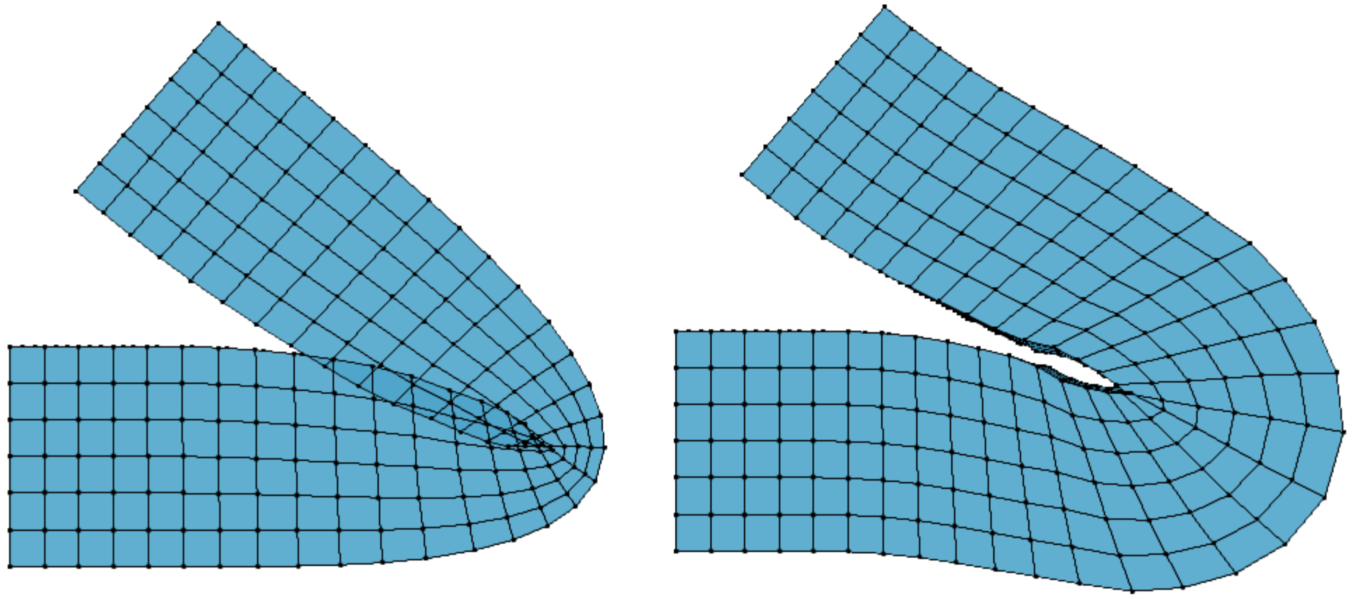
[Skinning with Dual Quaternions. Kavan et al. I3D, 2007]



LBS



DQS

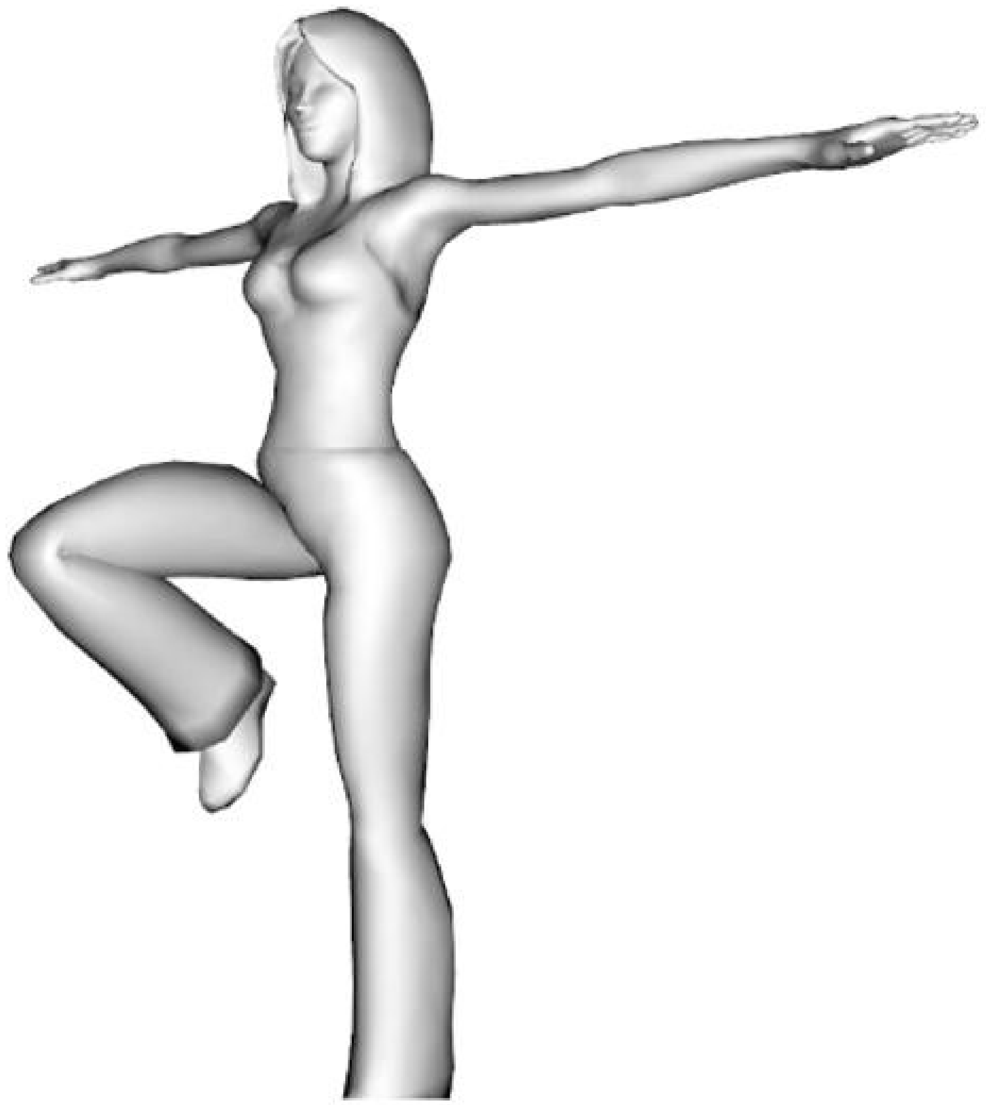


LBS DQS

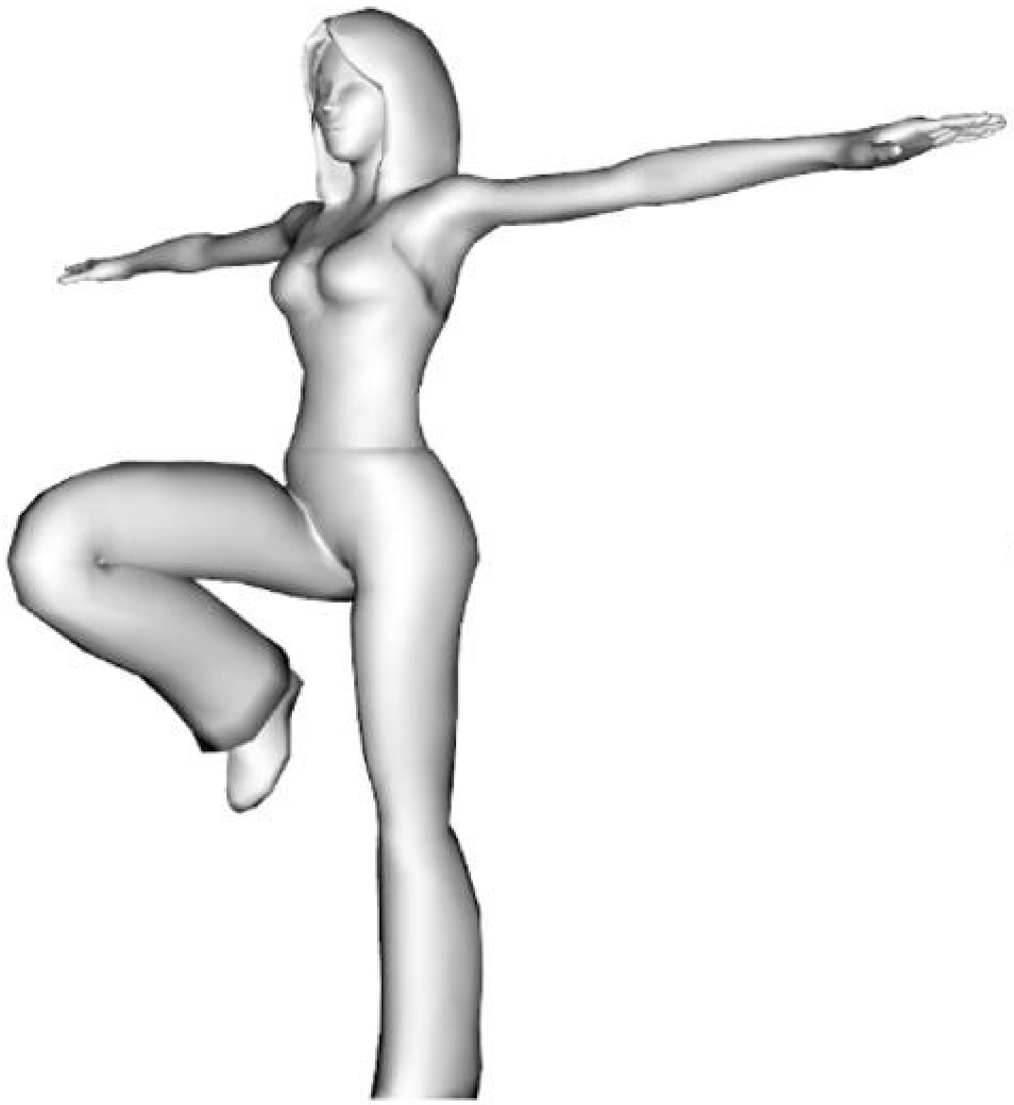
Dual Quaternion VS LBS

Dual quaternion

- (+) Fully solves *candy wrapper* artifact
- (+) Almost as efficient as LBS
- (-) May create artificial/unwanted bulge
  - *Not always prefered to LBS*
  - *Both solutions (LBS, DQS) are proposed in standard tools*

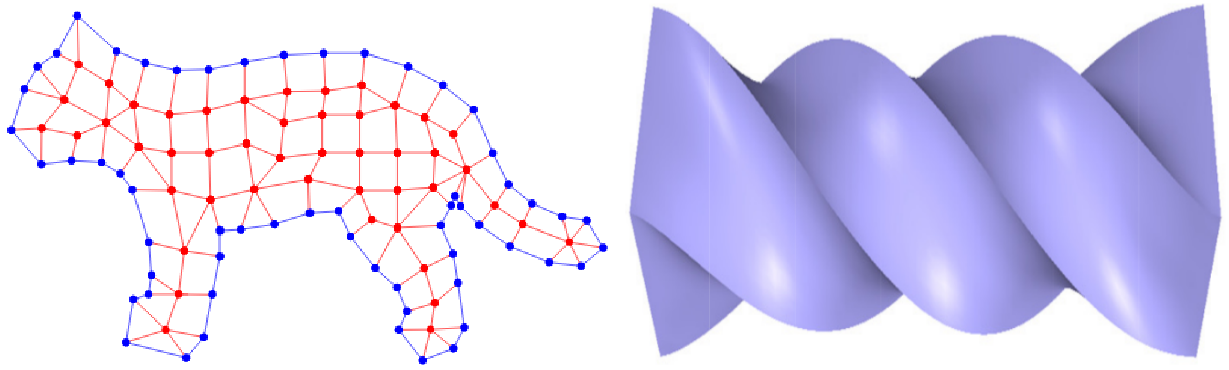


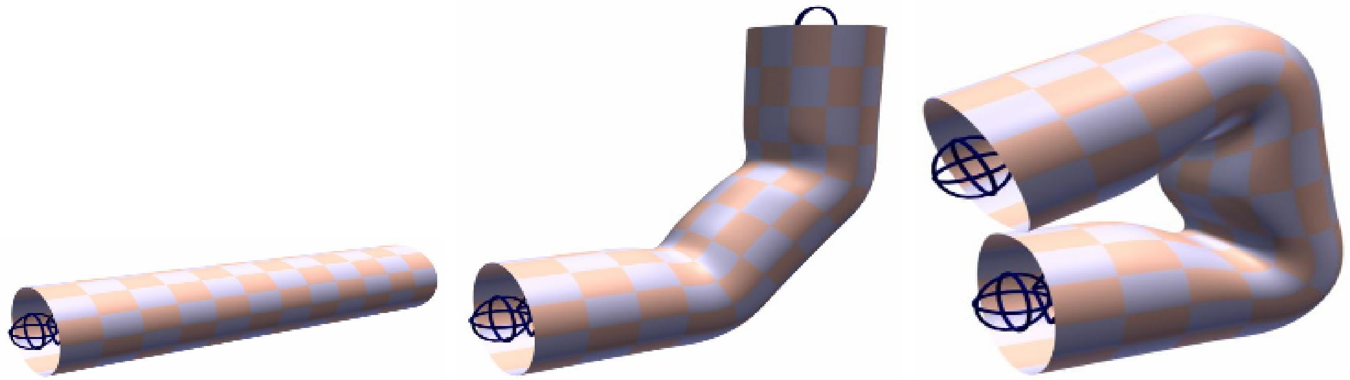
LBS

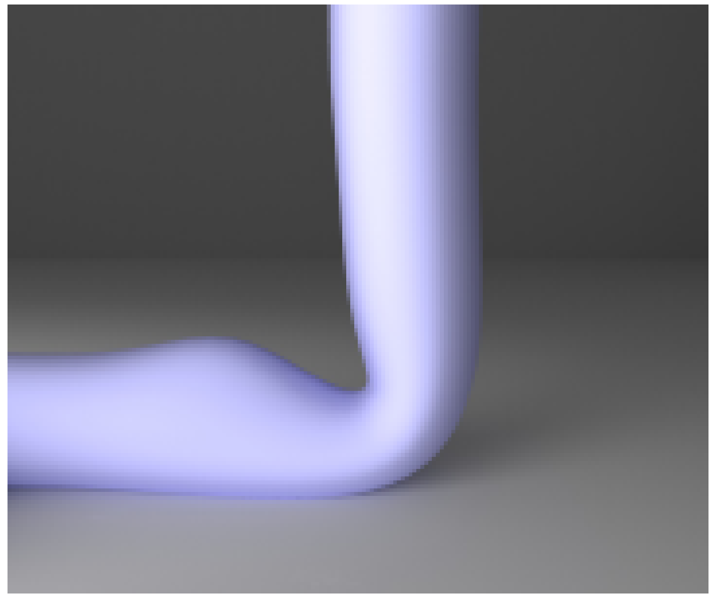


DQS

Extension - Volume preserving skinning







- Optimization based model using tetrahedrons

  [Large Mesh Deformation Using the Volumetric Graph Laplacien. K. Zhou et al. ACM SIGGRAPH 2005]

- Vector field based deformers

  [Kinodynamic skinning using volume-preserving deformations. A. Angelidis and K. Singh. SCA 2007]

- Direct geometrical volume preservation

  [Exact volume preserving skinning with shape control. D. Rohmer et al. SCA 2009]

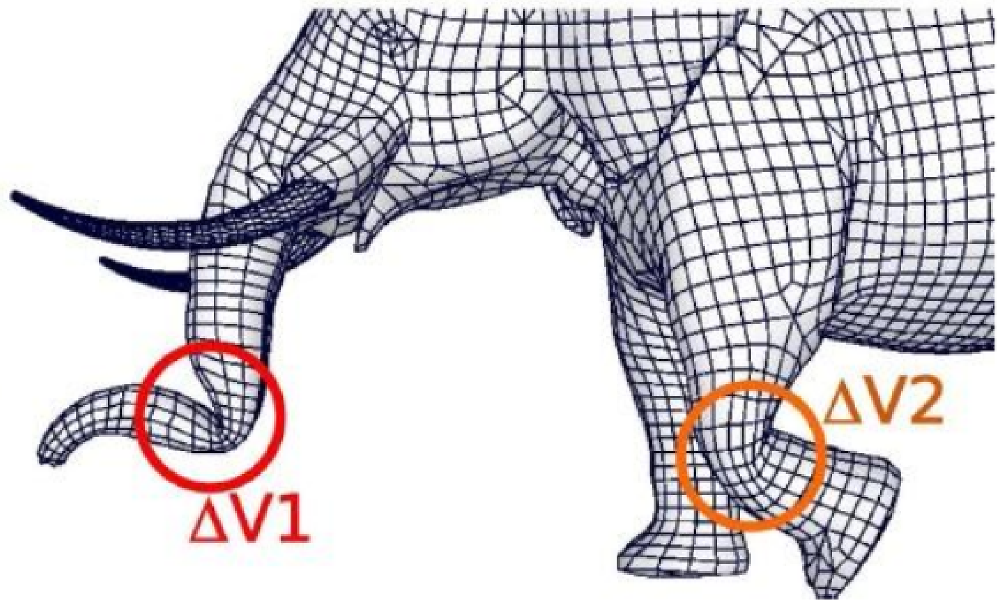
Direct geometrical volume preservation

Objective

Correct shape surface to compensate for the change of volume due to skinning deformation

Method

1. Apply standard skinning deformation (ex. LBS, DQS)

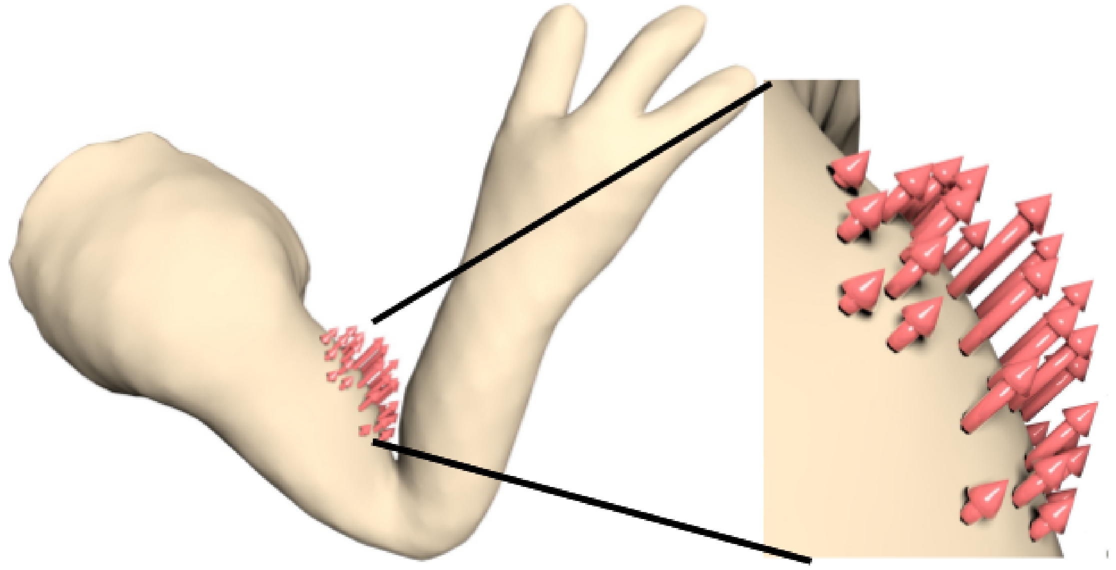


2. Compute the change of volume due to skinning deformation

   Sub-problem 1: how to compute the volume of a mesh
   Sub-problem 2: how to localize the change of volume

3. Deform the surface to reach the initial volume (or another target volume)

   Sub-problem 1: How to compute such deformation
   Sub-problem 2: How to parameterize it



Computing the volume delimited by a mesh

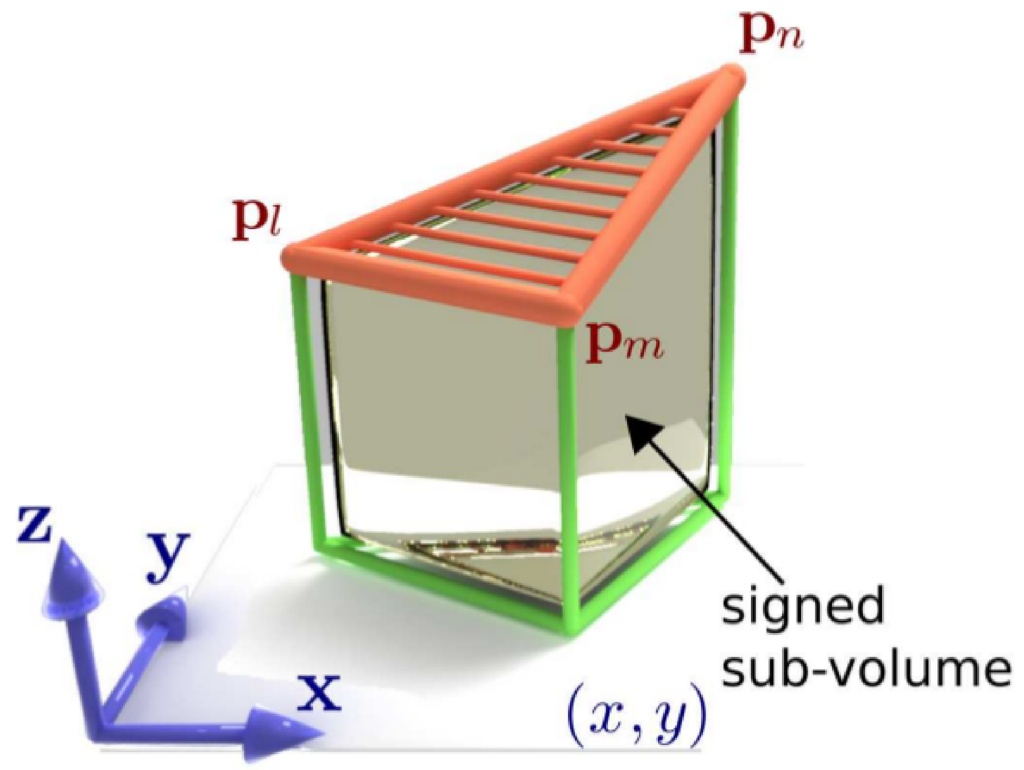
Analytical expression for a smooth surface S with coordinates $p = (p_x, p_y, p_z)$, and normal $n = (n_x, n_y, n_z)$

$$V = \int_\Omega \mathrm{d}\Omega = \int_S p_z\, n_z\, \mathrm{d}S$$

Use divergence theorem $\int_\Omega \mathrm{div}(F)\, \mathrm{d}\Omega = \int_S F \cdot n\, \mathrm{d}S$

With $F(p) = (p \cdot u_z)\, u_z\,,\; u_z = (0,0,1)\;\; (\mathrm{div}(F) = 1)$

$\Rightarrow V = \int_S (p \cdot u_z)\,(u_z \cdot n)\mathrm{d}S = \int_S p_z\, n_z \mathrm{d}S$



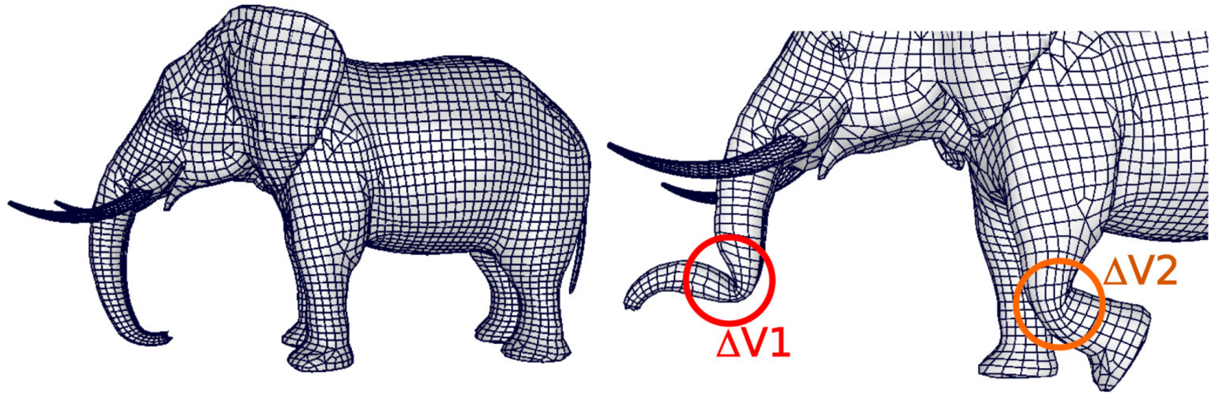
For a triangulated mesh

$$V = \sum_{\text{face } k} \int_{\text{triangle } t_k} p_z\, n_z\, \mathrm{d}S \text{ (on each triangle: } p_z \text{ is linear, } n_z \text{ is constant)}$$

...

$$V = \sum_{triangles\ (m,n,l)} \frac{z_m + z_n + z_l}{6} \begin{vmatrix} x_n - x_m & y_n - y_m \\ x_l - x_m & y_l - y_m \end{vmatrix} \qquad \text{(trilinear expression in } x, y, z))$$

Localizing the change of volume



1st solution

Segment the mesh into subpart

ex. Using skinning weights

Compute volume difference between each part

may need to close each part

(+) Fast

(-) Segmentation artifact

No overlaping influence b/w parts

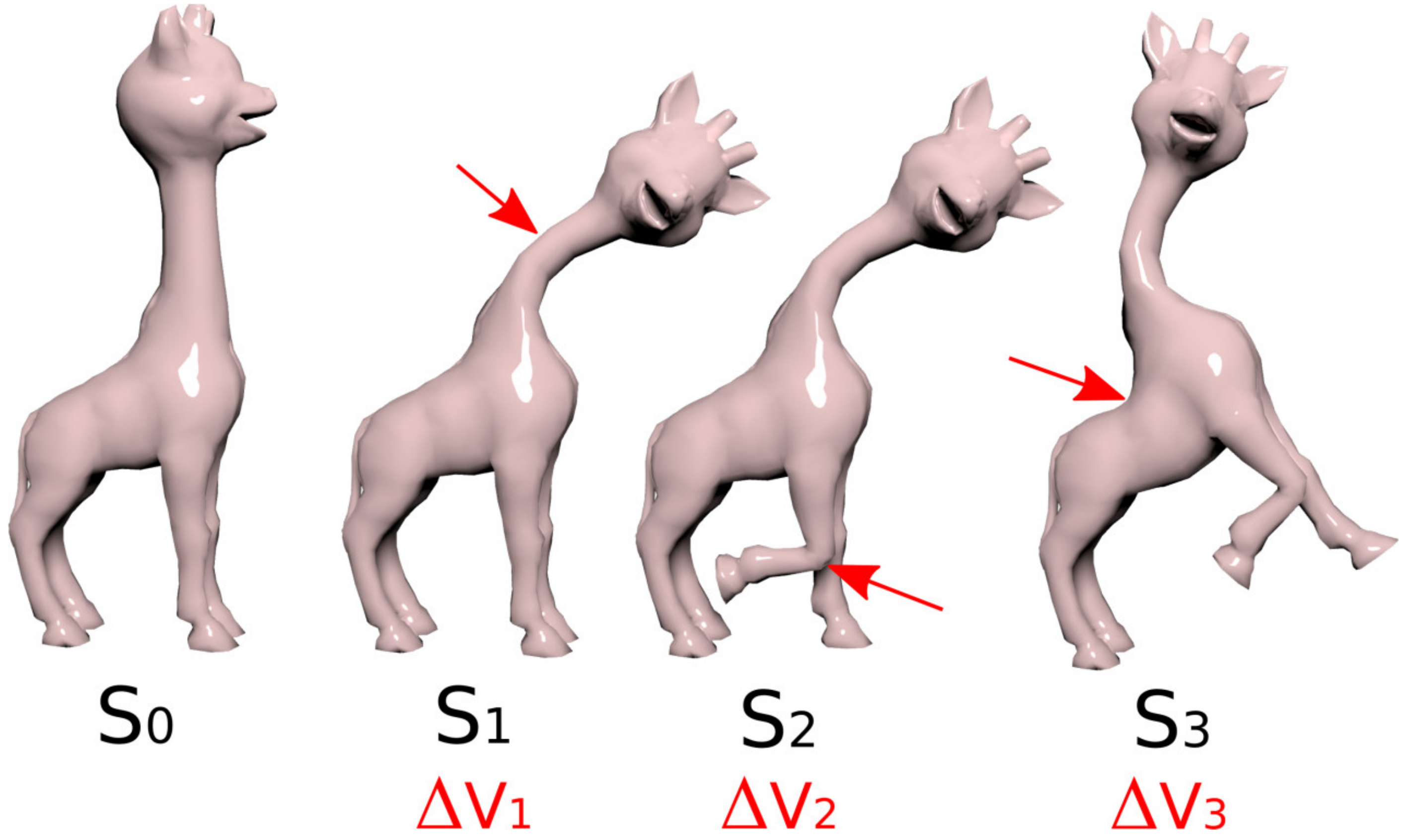
2nd solution

Compute the change of volume iteratively

For each joint articulation

(+) Artifact free

(-) Slower (iterate over all joints)



Compute deformation

Deform each vertex position p_k by u_k such that $V(p_k + u_k) = V_{target}$

Infinite number of solution: select the minimal deformation min $\sum_k \|u_k\|^2$

Allow some vertices to be more easy to deform using weight γ_k

$$\begin{cases} \min \displaystyle\sum_k \frac{\|u_k\|^2}{\gamma_k} \\ \text{subject to} \;\; V(p_k + u_k) = V_{target} \end{cases}$$

General method to solve constrained minimization: Lagrange multipliers

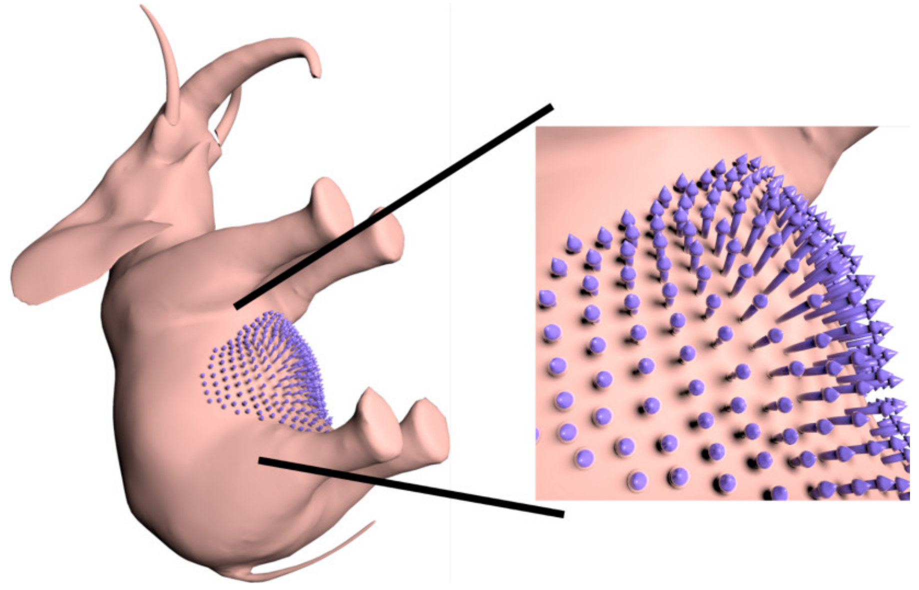
Energy to minimize: $E(u_k, \lambda) = \displaystyle\sum_k \frac{\|u_k\|^2}{\gamma_k} + \lambda \left(V(p_k + u_k) - V_{target} \right)$

Find (u_k, λ) such that $\nabla E = 0$

$V(p_k + u_k) - V_{target} = 0$

$\forall k \;,\;\; \frac{\partial E}{\partial p_k} = 0$

Deformation along normal direction



$u = \rho\, n$ with n unit normal ($\|n\| = 1$)

Energy to minimize: $E = \|\rho_k\|^2/\gamma_k + \lambda(V((p_k + \rho_k n_k)_k) - V_{target})$

Non linear constraint $\rightarrow$ Linearize assuming small deformation $|\rho_k| \ll 1$

$$V((p_k + \rho_k n_k)_k) \simeq V(p_k) + \sum_k \rho_k\, n_k \cdot \nabla V(p_k)$$

$$\Rightarrow V((p_k + \rho_k n_k)_k) - V_{target} = \sum_k \rho_k\, n_k \cdot \nabla V(p_k) + \underbrace{(V(p_k) - V_{target})}_{\Delta V}$$

$$E = \sum_k \|\rho_k\|^2/\gamma_k + \lambda(\sum_k \rho_k\, n_k \cdot \nabla V(p_k) - \Delta V)$$

$$\nabla E = 0 \Rightarrow \begin{cases} \forall k,\ 2\rho_k/\gamma_k + \lambda n_k \cdot \nabla V(p_k) = 0 & (1) \\ \sum_k \rho_k n_k \cdot \nabla V(p_k) = \Delta V & (2) \end{cases}$$

Extract p_k from (1), insert to (2) $\Rightarrow \lambda = -2\Delta V / \sum_k \gamma_k\ (n_k \cdot \nabla V(p_k))^2$

$$\Rightarrow \rho_k = \Delta V \frac{\gamma_k\ n_k \cdot \nabla V(p_k)}{\sum_j \gamma_j (n_j \cdot \nabla V(p_j))^2}$$

Deformation along three orthogonal direction

Use the trilinear property of the volume

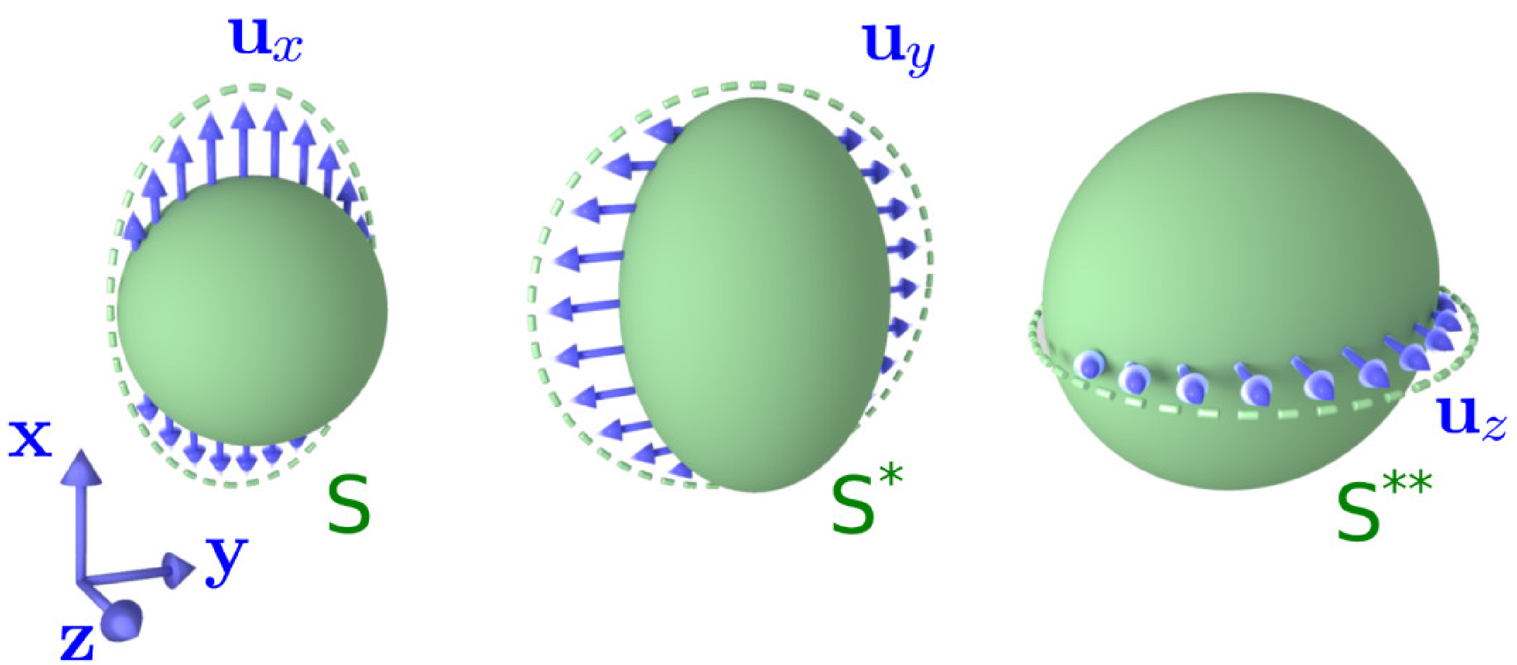
1. Solve for $\Delta V/3$ in x direction (y, z are constant $\Rightarrow$ linear)
2. Then $\Delta V/3$ in y direction (x, z are constant $\Rightarrow$ linear)
3. Finally $\Delta V/3$ in z direction (x, y are constant $\Rightarrow$ linear)

$$u_k = \gamma_k \frac{\Delta V}{3} \left(\frac{\nabla_x V(p_k)}{\sum_j \gamma_j (\nabla_j V(p_j))^2}, \frac{\nabla_y V^\star(p_k)}{\sum_j \gamma_j (\nabla_j V^\star(p_j))^2}, \frac{\nabla_z V^{\star\star}(p_k)}{\sum_j \gamma_j (\nabla_z V^{\star\star}(p_j))^2} \right)$$

Need to update the gradient at each iteration:

$V^\star$ after correction along x

$V^{\star\star}$ after correction along y



(+) Exact volume correction

(-) Depends on the order/choice of correction in each axis.

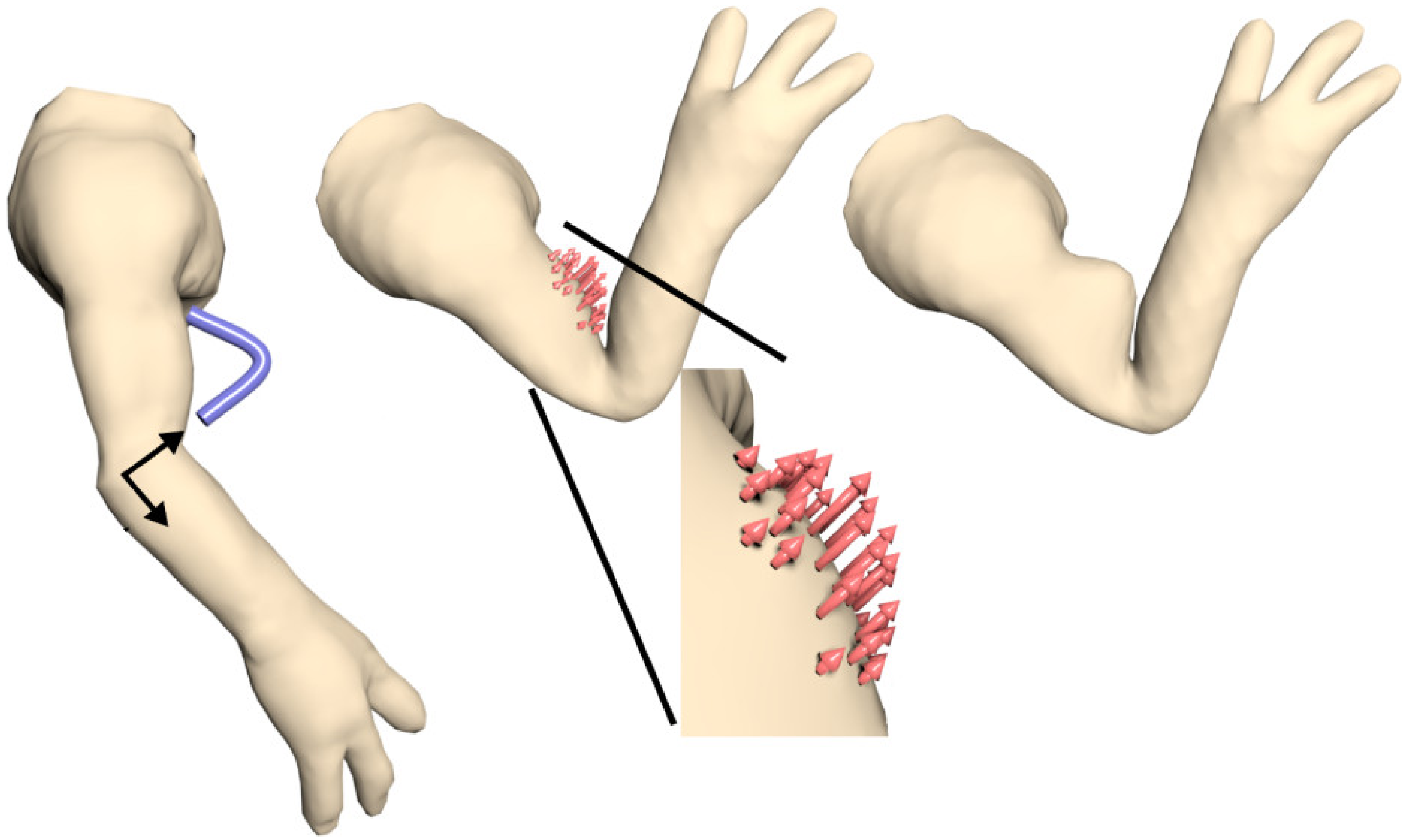
Controling the deformation

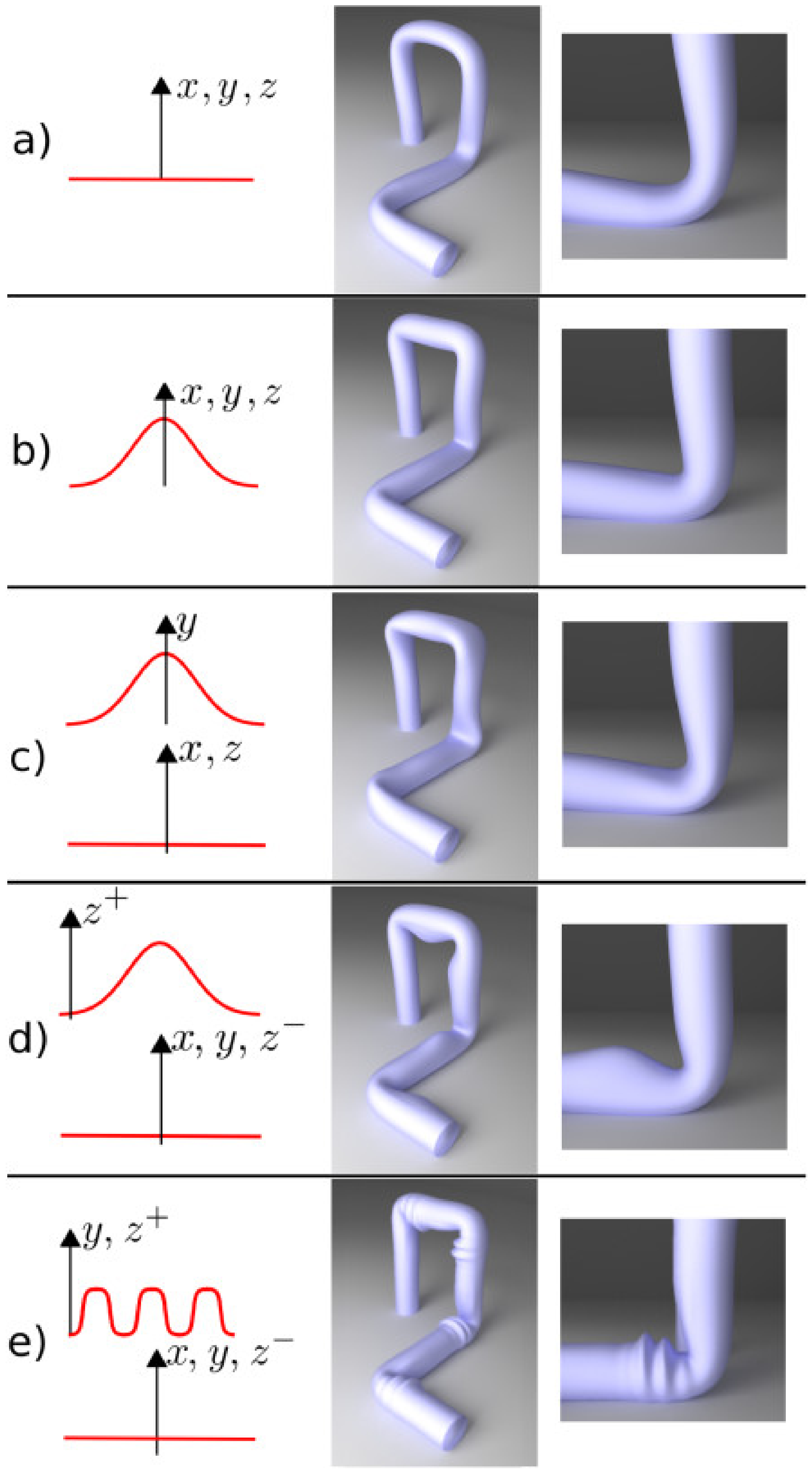
Control on the weights distribution ω_k

- Use profile curve

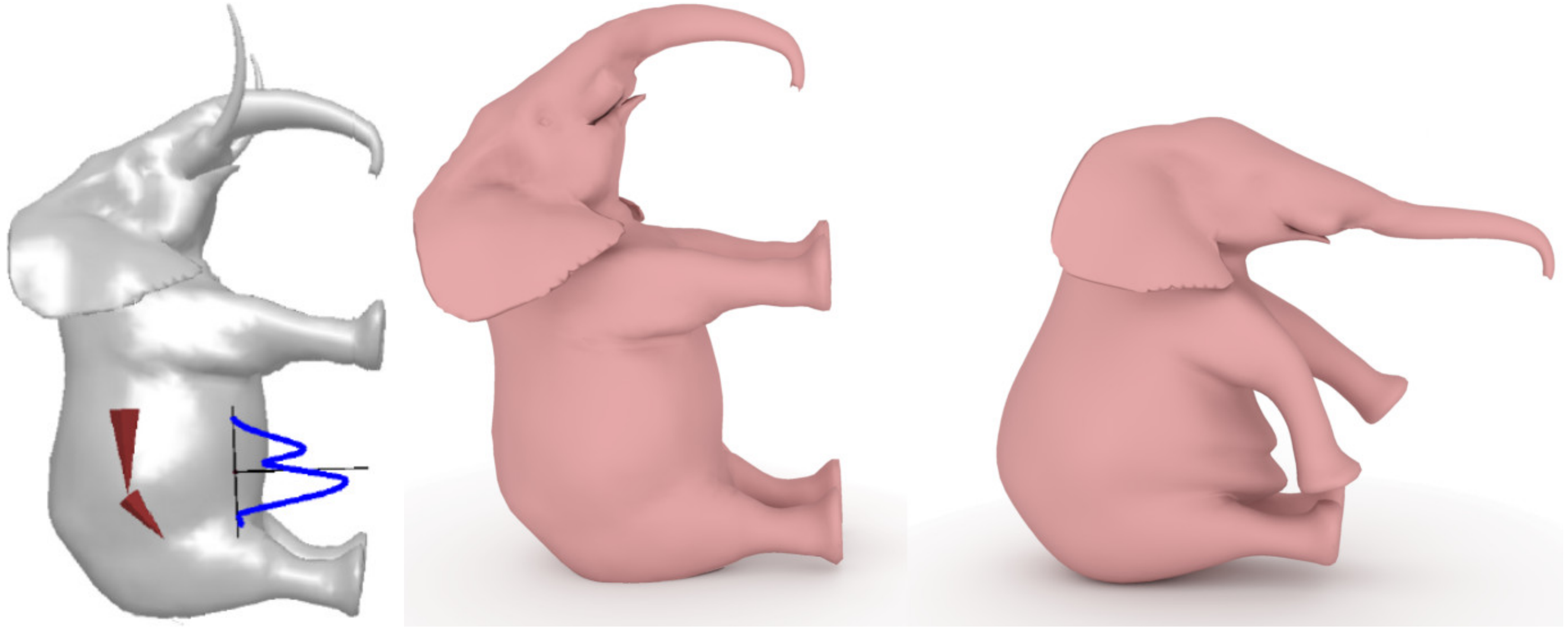
  Defined in the local frame of the joint/bone

- Weighted by Gaussian distribution in space





Results



[Exact volume preserving skinning with shape control. D. Rohmer et al. SCA 2009]
[Local Volume Preservation for Skinned Characters. D. Rohmer et al. PG 2008]

Brute force physically-based modeling

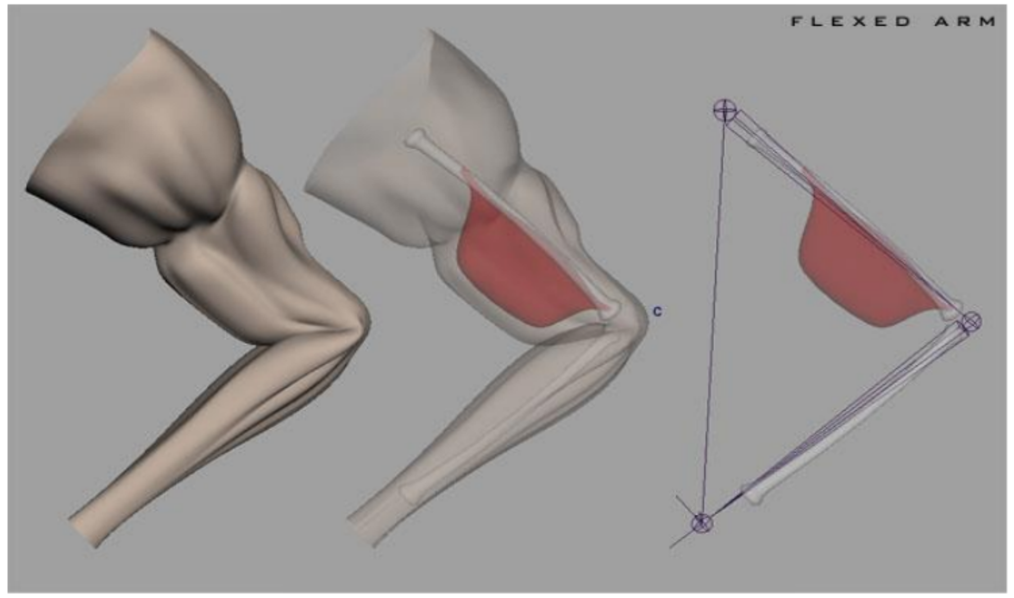
- Model muscles attached to bones
- Simulate muscle deformation using FEM
- Add an elastic skin layer on top of muscles

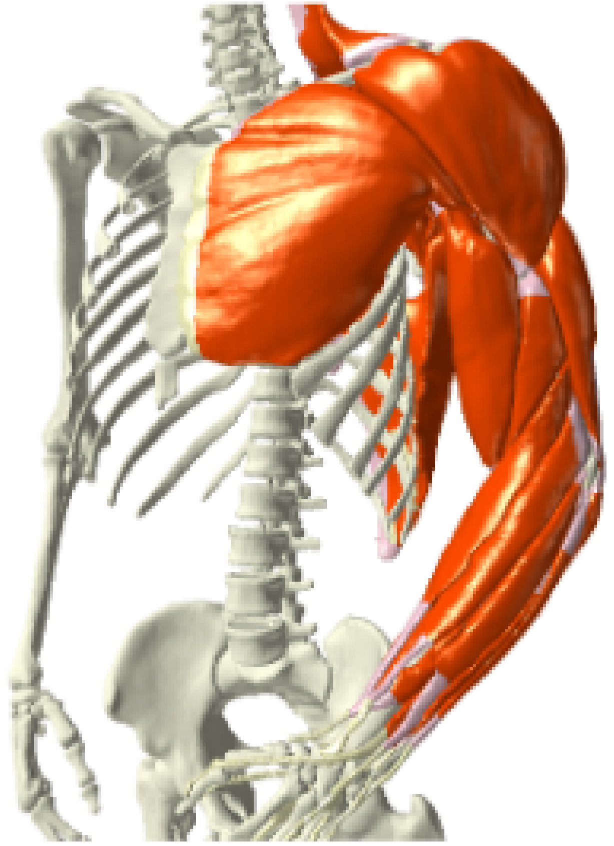
(+) Detailed dynamic results

(-) Computational time

(-) Teadious to setup and control

[Teran et al., Creating and Simulating Skeletal Muscle from the Visible Human Data Set. IEEE TVCG 2005]





30 muscles, 10×10^6 *tetrahedrons*

0:00

Character Animation

Skeletal Animation

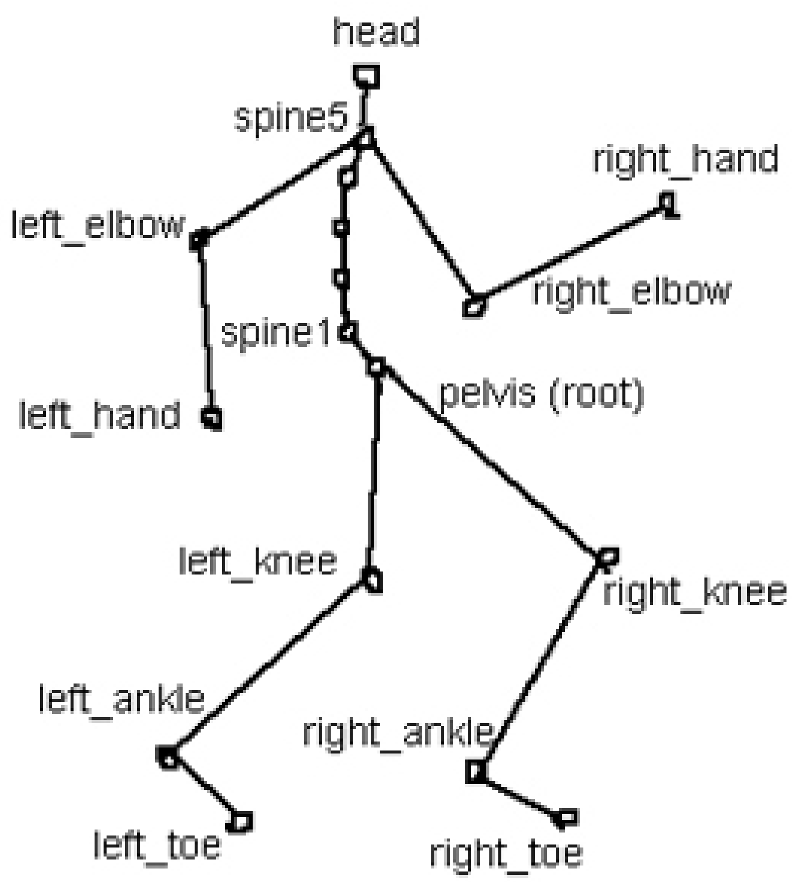
Skeleton structure

Characteristics

- Hierarchical representation

  All children follow the transformation of the parent

  Need to define a root: usually at the hips/pelvis



- Convenient to express local deformation with respect to the parent

  ex. Rotate knee from 20°

Converting local to global frames/joint coordinates

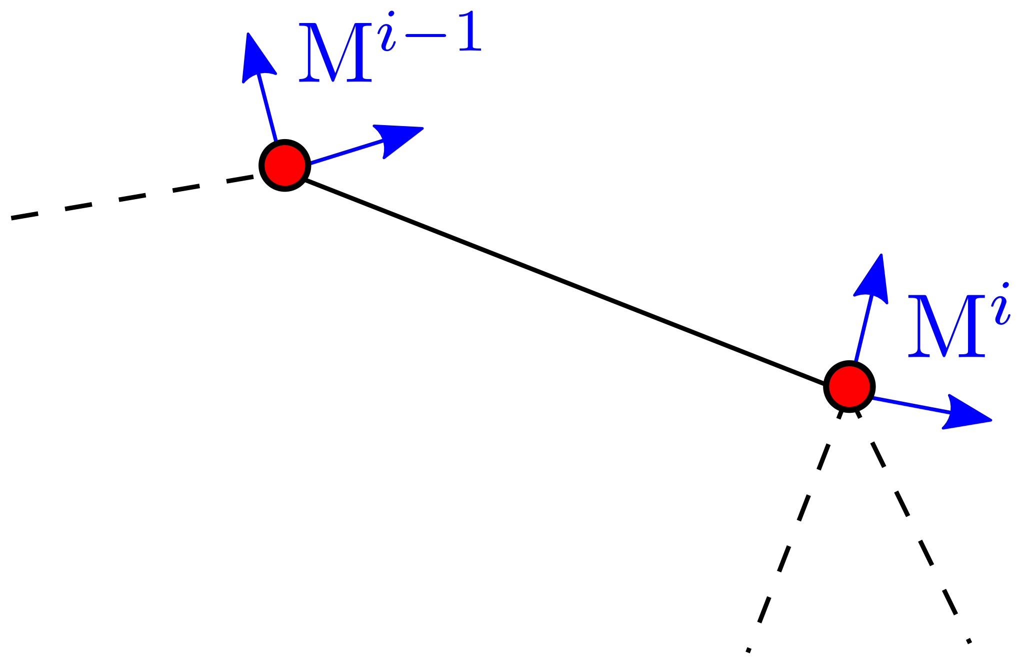
- With 4x4 matrices M

$$M^i_{global} = M^{i-1}_{global}\ M^i_{local}$$

- With translation t, rotation R

$$R^i_{global} = R^{i-1}_{global}\ R^i_{local}$$

$$t^i_{global} = t^{i-1}_{global} + R^{i-1}_{global}\ t^i_{local}$$



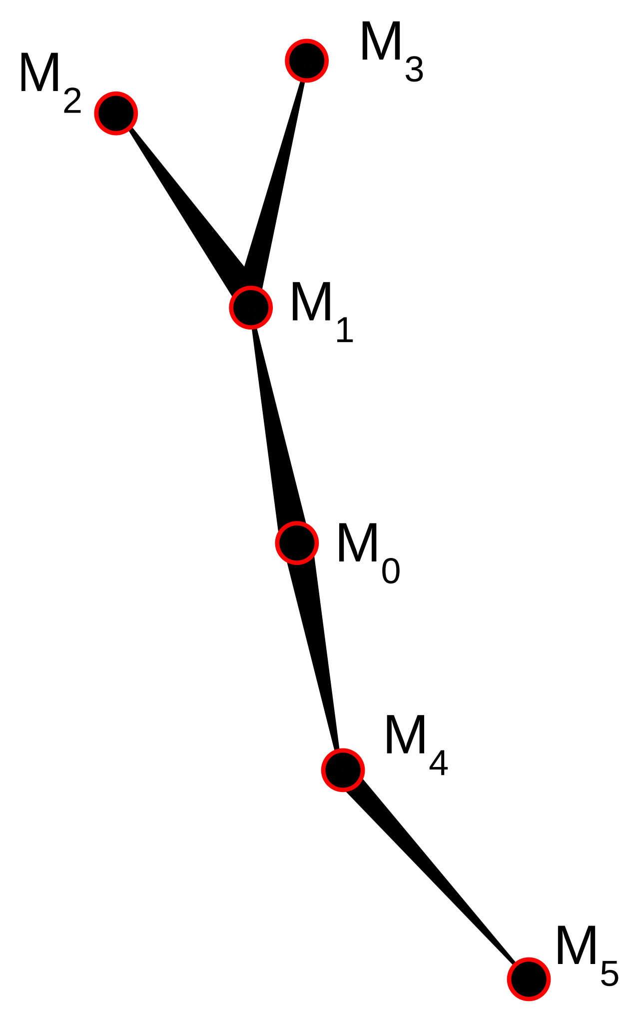
Encoding hierarchical skeleton

- Simplest encoding based on index within vector

```
Geometry=[M0, M1, M2, M3, M4, M5]
Parent = [-1,0,1,1,0,4]
```

- Convert local coordinates to global coordinates

```
local (Geometry)  <- std::vector of rotation (r), translation (p)
global (Geometry) <- std::vector of rotation (r), translation (p)
global[0] = local[0];
for(size_t k=1; k<N; ++k)
{
    int parent = Parent[k];
    global[k].r = global[parent].r * local[k].r;
    global[k].p = global[parent].r * local[k].p + global[parent].p;
}
```

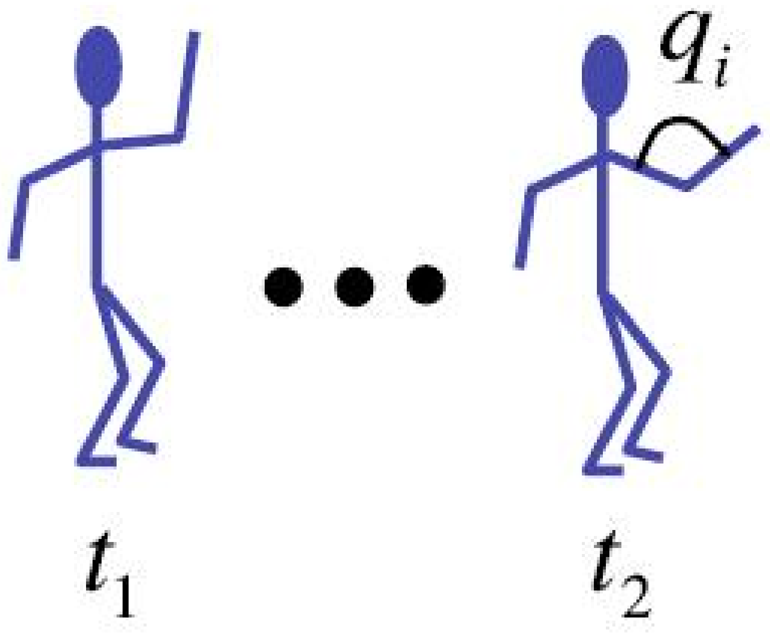


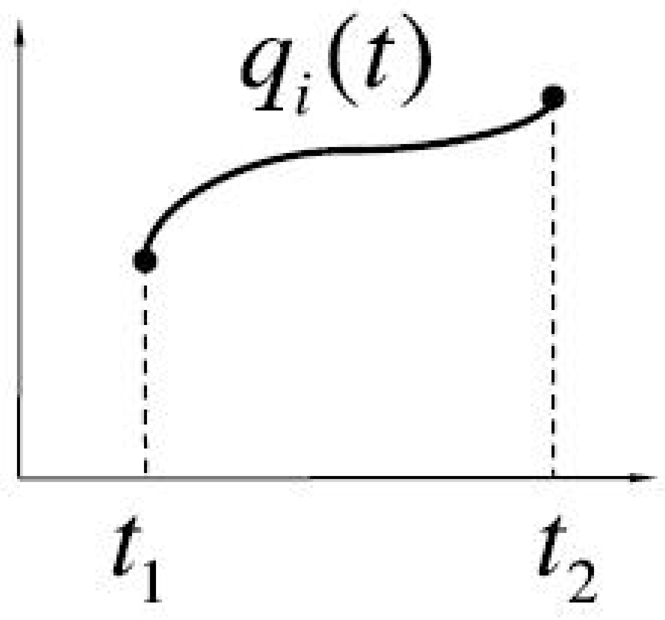
Forward kinematics

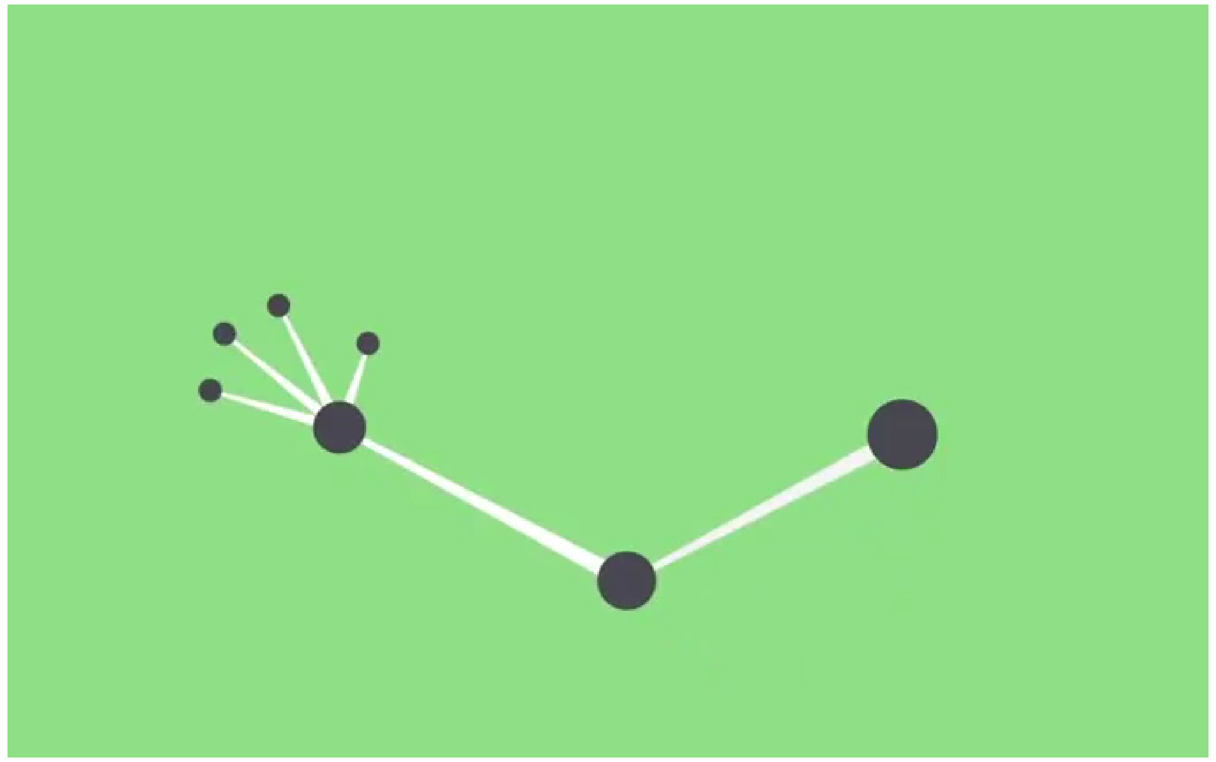
FK - **F**orward **K**inematics

- Each join angle is set manually
- Adapted to set orientation of specific parts
- Interpolate rotations during animation

(+) Generates curved trajectory naturally



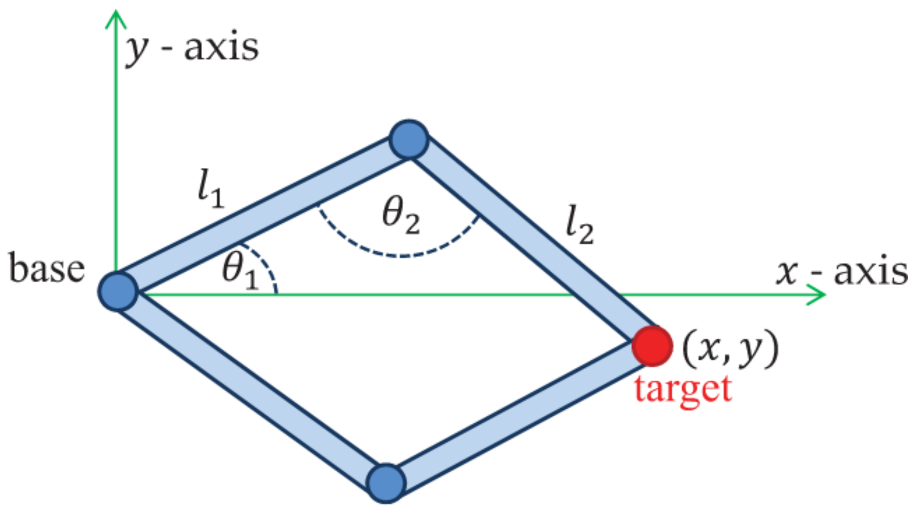




MiloScerny Animation

Inverse Kinematics

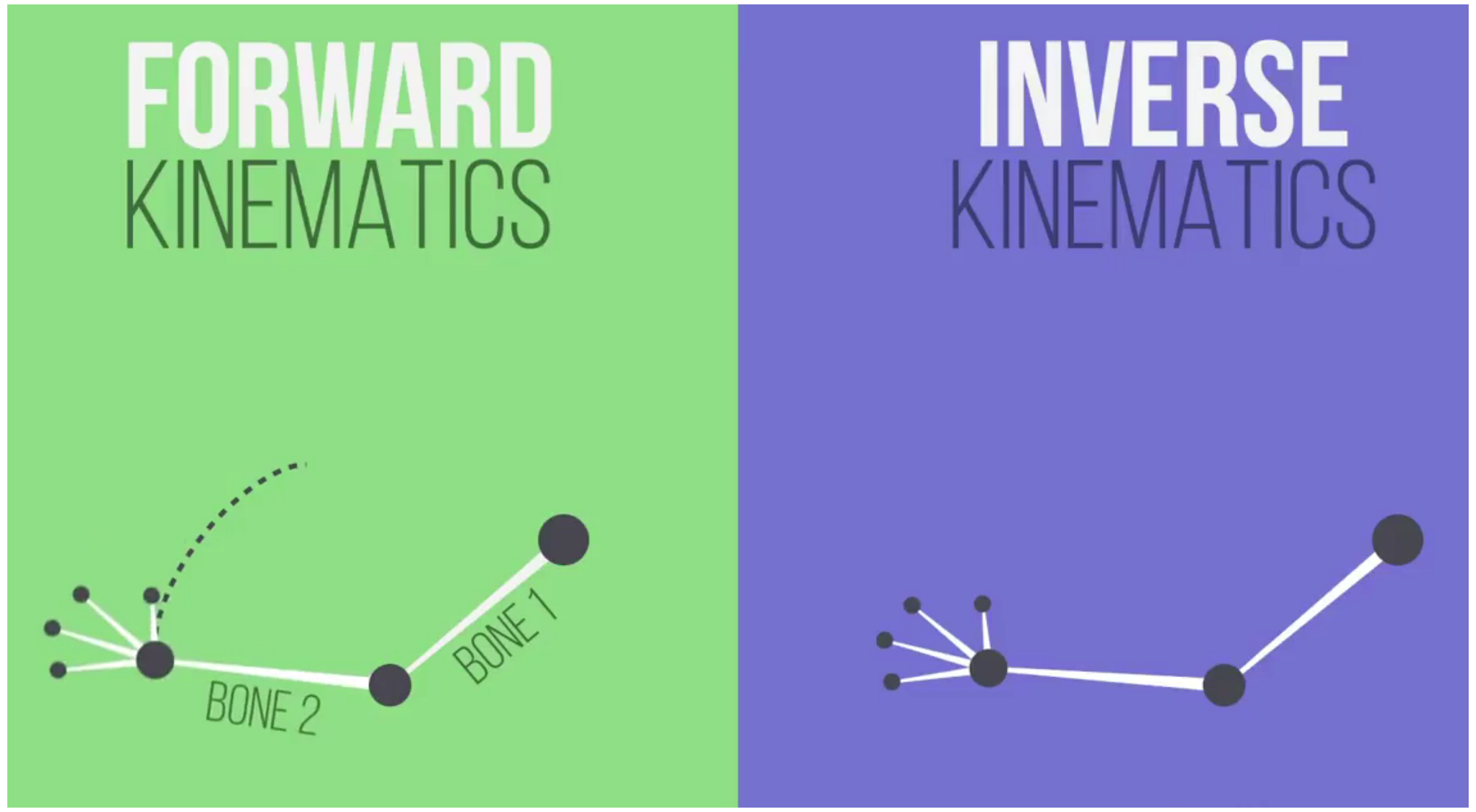
IK: **I**nverse **K**inematics



- Describe position (/and orientation) of *end-effectors* (contact, walking, etc)
- Compute joint angles reaching this position

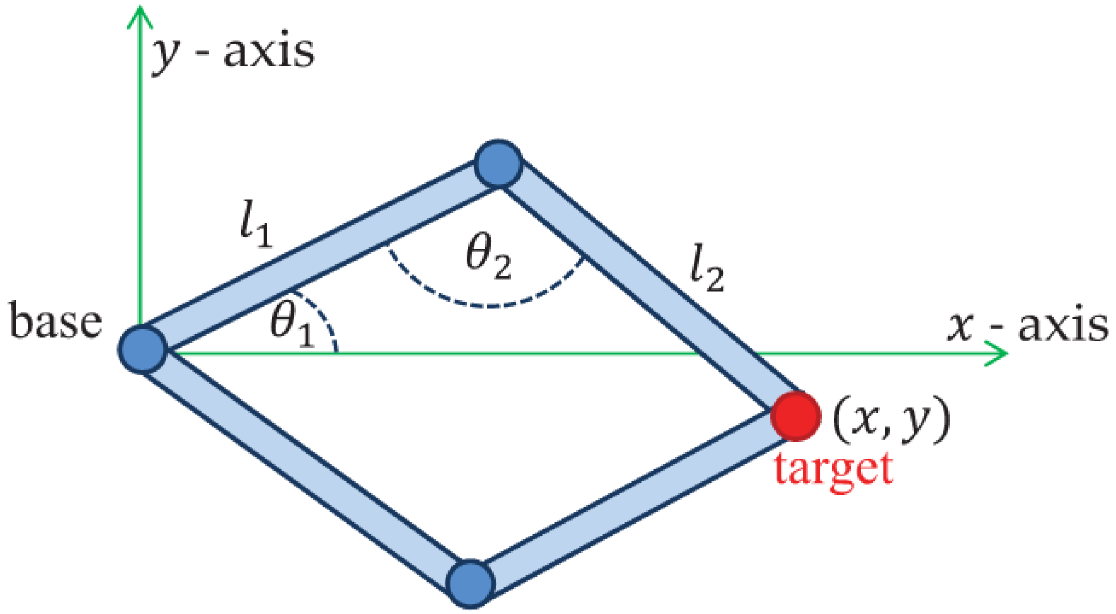
$$p_k = p_0 + \sum_{i=0}^{k-1} l_i \; R_i \, u_i$$

R_i can be expressed with various rotation parameters (Matrices, Euler angles, axis/angle, quaternions, etc)



MiloScerny Animation

IK Example with two bones



[Inverse Kinematics Techniques in Computer Graphics: A Survey. A. Aristidou et al. STAR EG. 2017.]

In general the general case $p_k = f(\theta_i)$

- Look for $\theta_i = f^{-1}(p_k)$
- f is a non linear function
- There may exists multiple solutions (or none)
- Solutions may exhibits discontinuities
- Closed form solution are not available

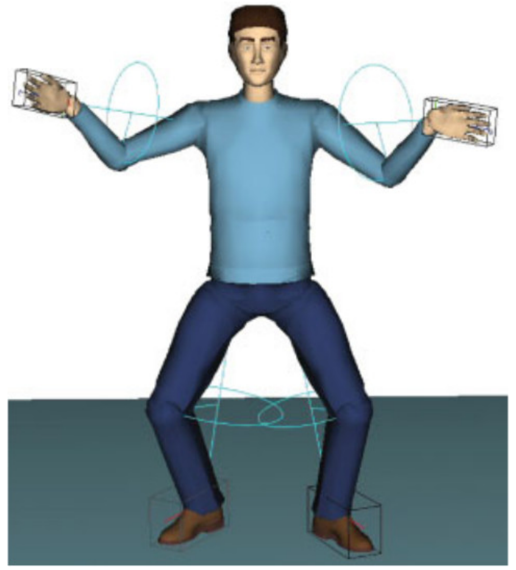
Two solutions defined by

$$\cos(\theta_1) = \frac{l_1^2 + x^2 + y^2 - l_2^2}{2l_1\sqrt{x^2+y^2}}$$

$$\cos(\theta_2) = \frac{l_1^2 + l_2^2 - (x^2 + y^2)}{2l_1 l_2}$$

Some attempts for explicit solutions in specific cases

[Real-Time Inverse Kinematics Techniques for Anthropomorphic Limbs. D. Tolani et al. Graphical Models, 2000.]
[Analytical inverse kinematics with bodyposture control. M. Kallmann. Comp. Anim. & Virt. Worlds, 2008]



IK: Numerical methods

Numerical inversion of $p = f(\theta), \theta = (\theta_0, \cdots, \theta_{N-1})$.

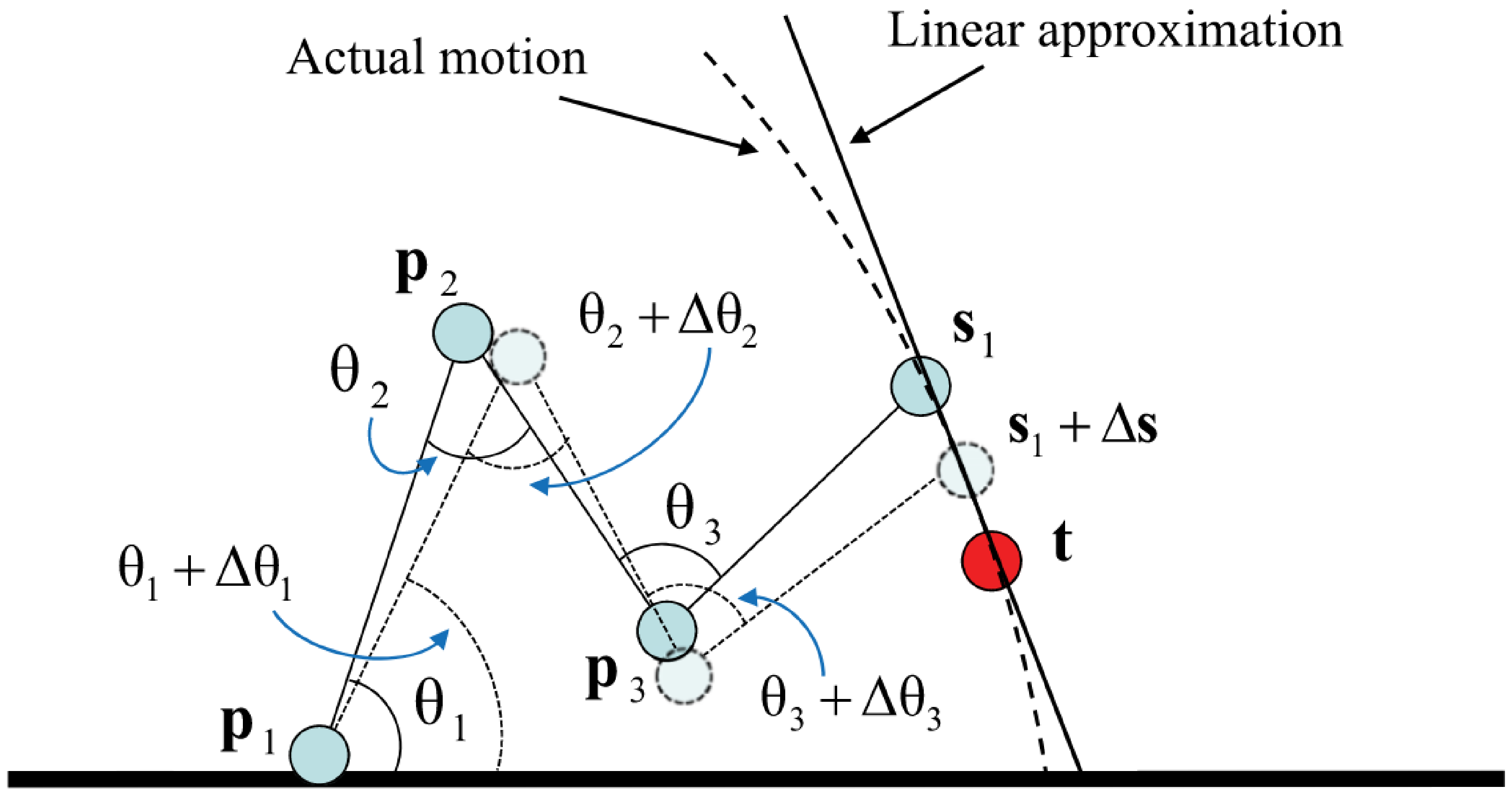
Consider small step size $p \to p + \Delta p$

$$\Delta p \simeq \underbrace{\left(\frac{\partial f}{\partial \theta}\right)}_{\mathrm{J}} \Delta\theta$$

J - Jacobian matrix.

$\to$ Not square $(3 \times N)$, not invertible.

Unknown > # constraints



IK: Numerical methods

Several possible approaches to solve $\mathrm{J}\,\delta\theta = \delta p$

- Pseudo Inverse

  $\Delta\theta = J^+\ \Delta p$, with $\mathrm{J}\mathrm{J}^+ = \mathrm{I}$

  $\mathrm{J}^+ = \mathrm{J}^T \left(\mathrm{J}\,\mathrm{J}^T\right)^{-1}$

- Can also be computed using SVD: $\mathrm{J}^+ = \mathrm{V}\Sigma^+\,\mathrm{U}^T$

  $\Sigma_{ii} = \sigma_i$, $\Sigma^+_{ii} = 1/\sigma_i$ if $\sigma_i \neq 0$, 0 otherwise.

- Adding damping to compensate for singularities

  $\Delta\theta = \mathrm{J}^T \left(\mathrm{J}\,\mathrm{J}^T + \lambda^2\,\mathrm{I}\right)^{-1}\ \Delta p$

- Using Newton's methods

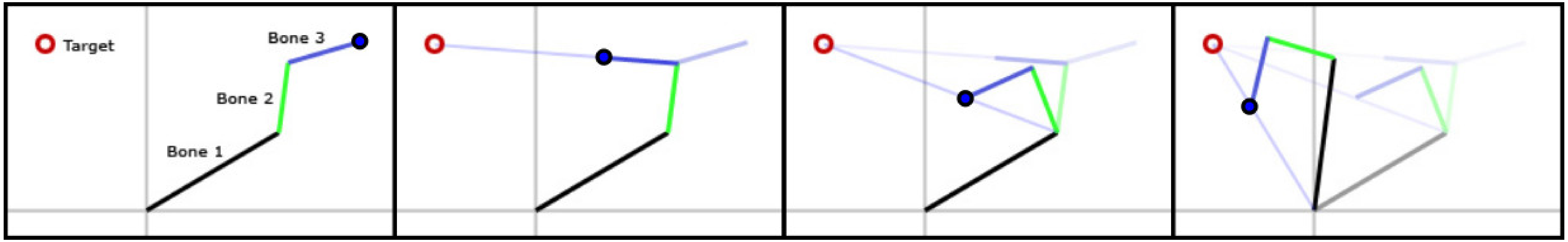
[Inverse Kinematics Techniques in Computer Graphics: A Survey. A. Aristidou. STAR EG 2017]

IK: Heuristic approach

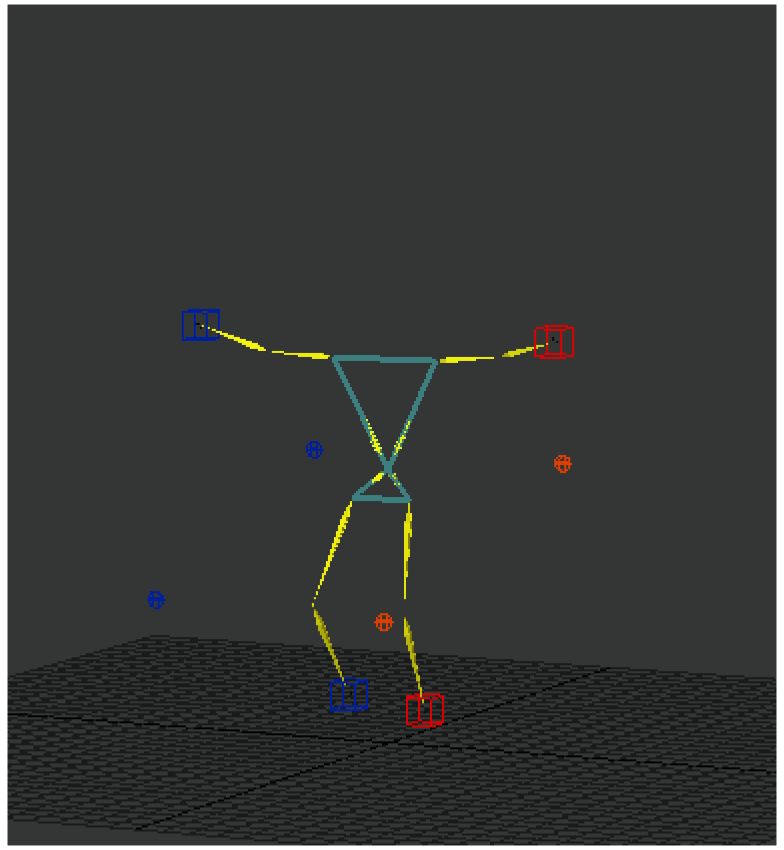
Cyclic Coordinates Descent (CCD)

- Iteratively rotates joint $j^N \rightarrow j^{i-1} \rightarrow \cdots \rightarrow j^1$ for the extremity (end effector) to be as close as possible from the target.
  = *End-effector aligned with the segment (joint,target)*
- Restart until convergence

[Making Kine More Flexible. Jeff Lander. Game Dev, 1998.]



IK: Heuristic approach

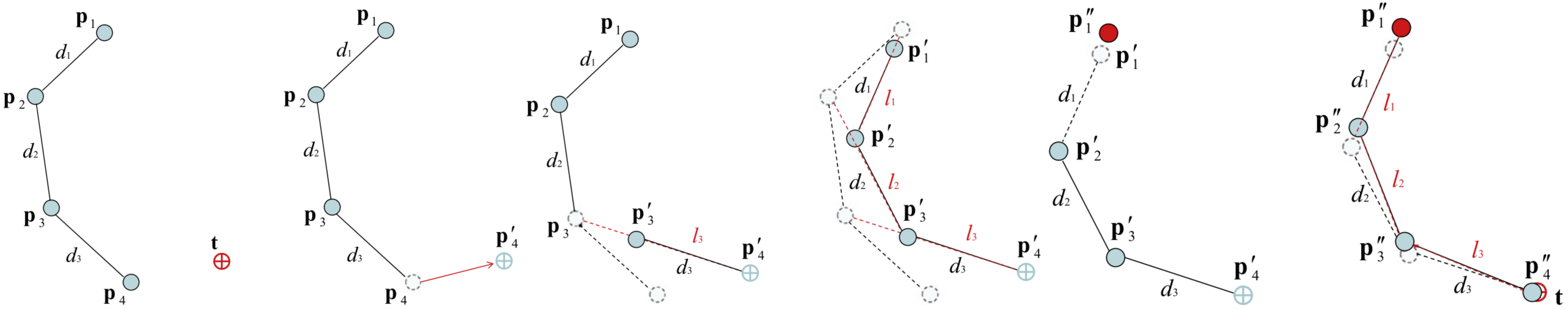


Fabrik

Iterate between

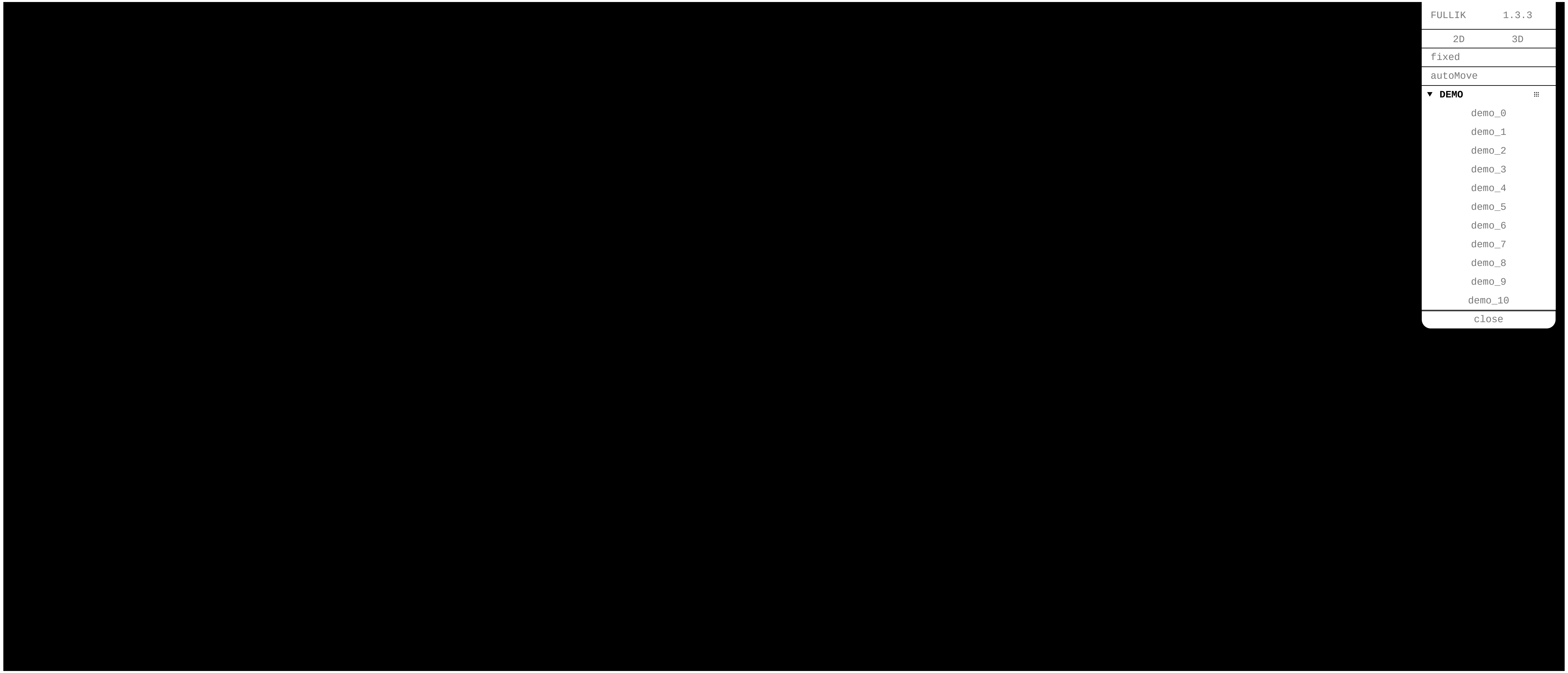
- Forward direction: Match the end-effector target
  propagate changes toward previous position to match bones' length.
- Backward direction: Match the starting position
  propagate changes toward following positions to match bones' length.

[A fast iterative solver for the Inverse Kinematics problem. A. Aristidou. Graphical Models 2011.]



Inverse Kinematics

Example



Synthesizing and controling skeleton animation

Blending skeleton animation

- Pre-store several looping animation
- Blend between animation for transition

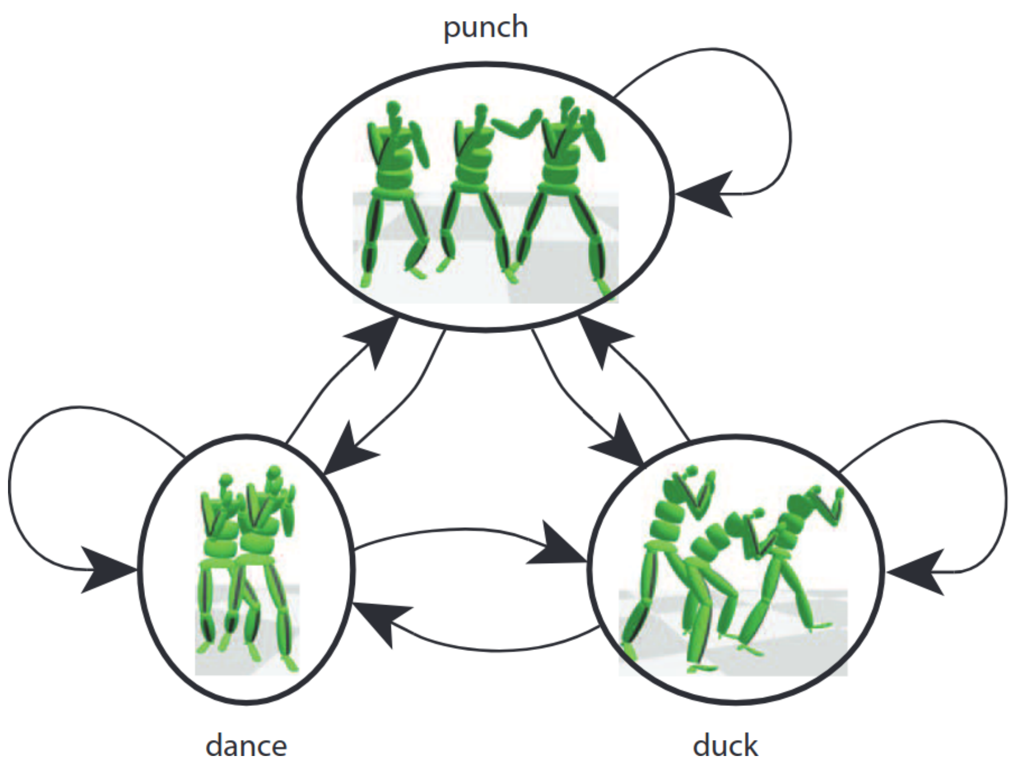
Motion graphs

Also called Move Trees (highly used in video games)

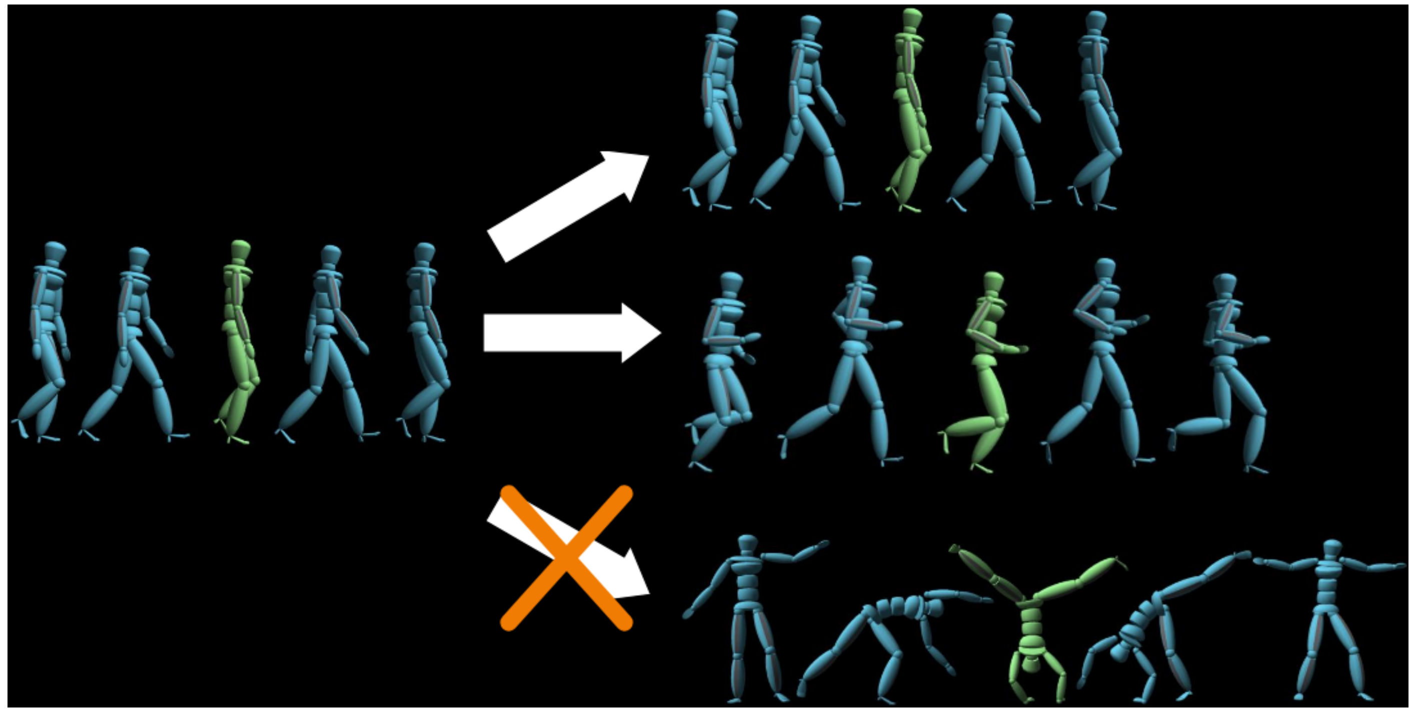
- Stores multiple precomputed animation

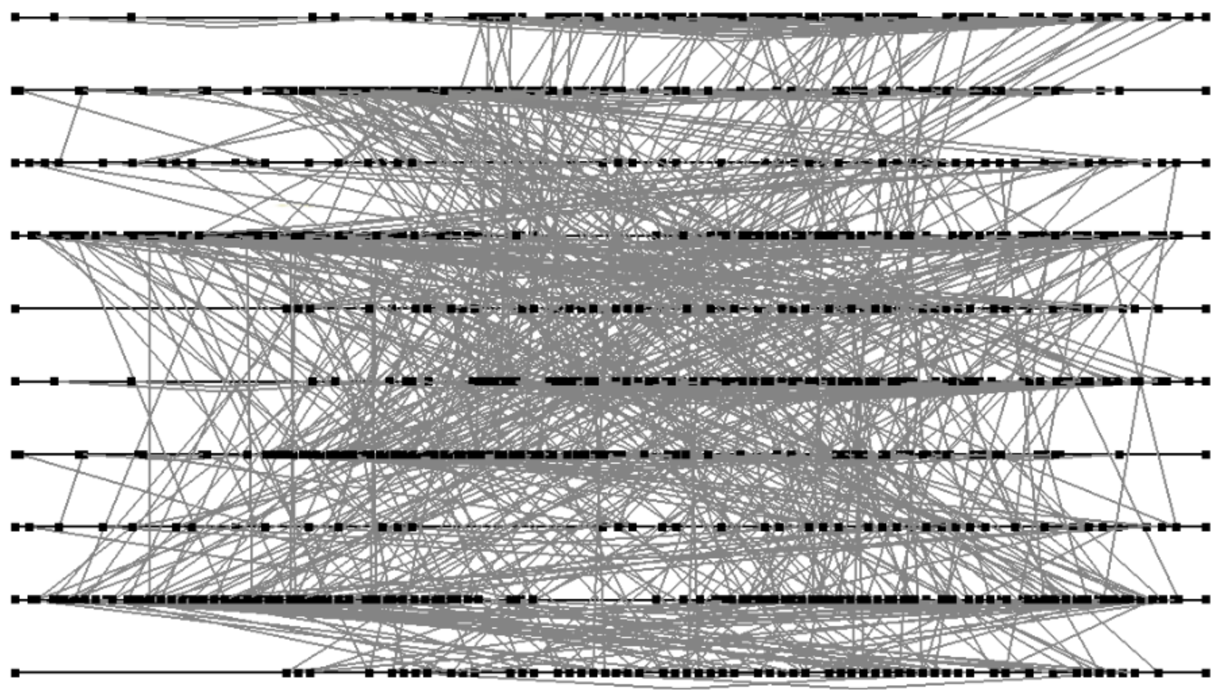
  Manually design, motion capture, etc

- Find optimal transitions between different motions



- *[Mizuguchi et al., Data driven motion transitions for interactive games, EG short paper, 2001]*
- *[Kovar et al., Motion Graphs, ACM SIGGRAPH 2002]*
- *[Heck and Gleicher, Parametric Motion Graphs, ACM SIGGRAPH 2007]*





Controlers

Mix between predefined motions and physics $\rightarrow$ allow user perturbations

1 - Define target $(\theta_{target}(t), \theta'_{target}(t))$

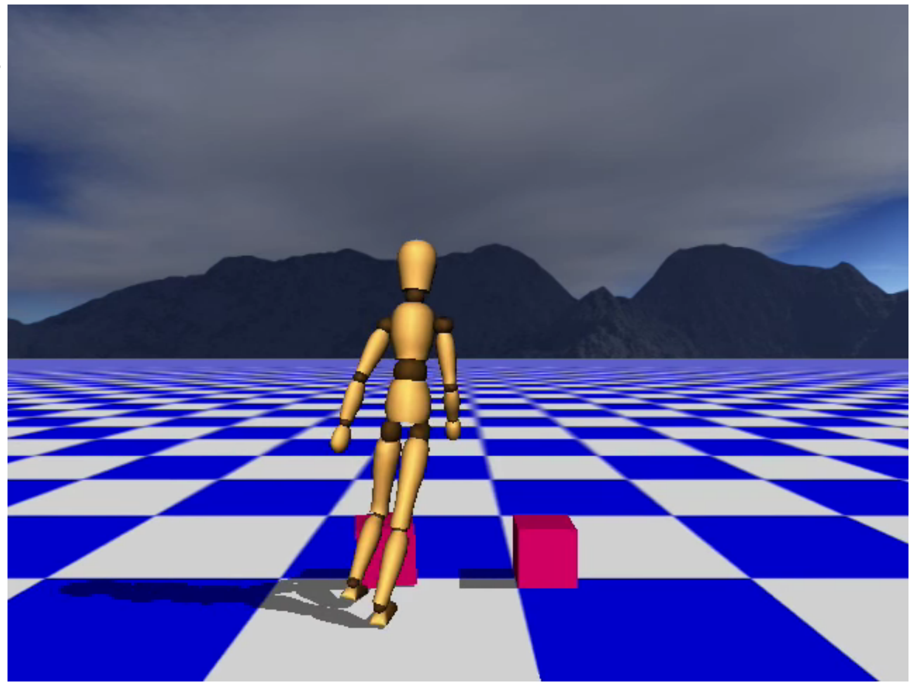
pre-defined finite state machine (Gait model)

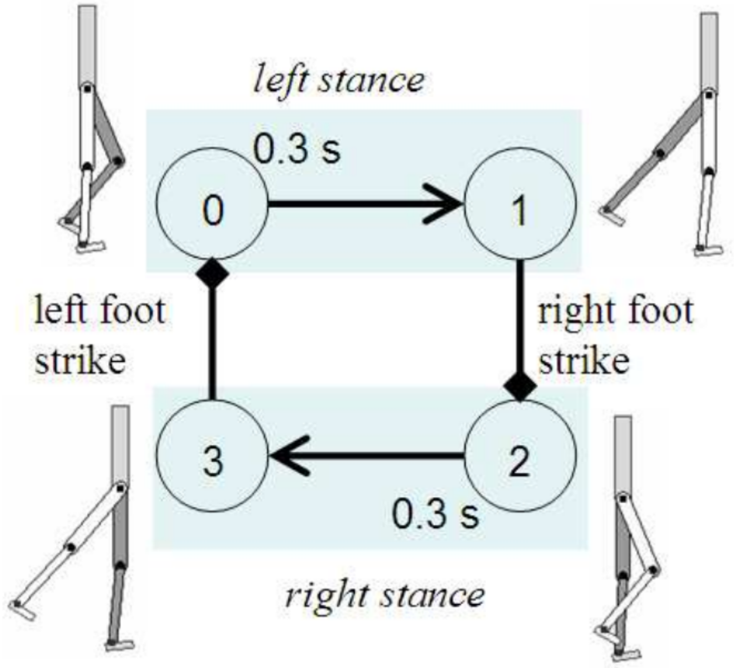
2 - Add user perturbation to the current state

3 - Use proportional derivative controlers to compute joint torque τ

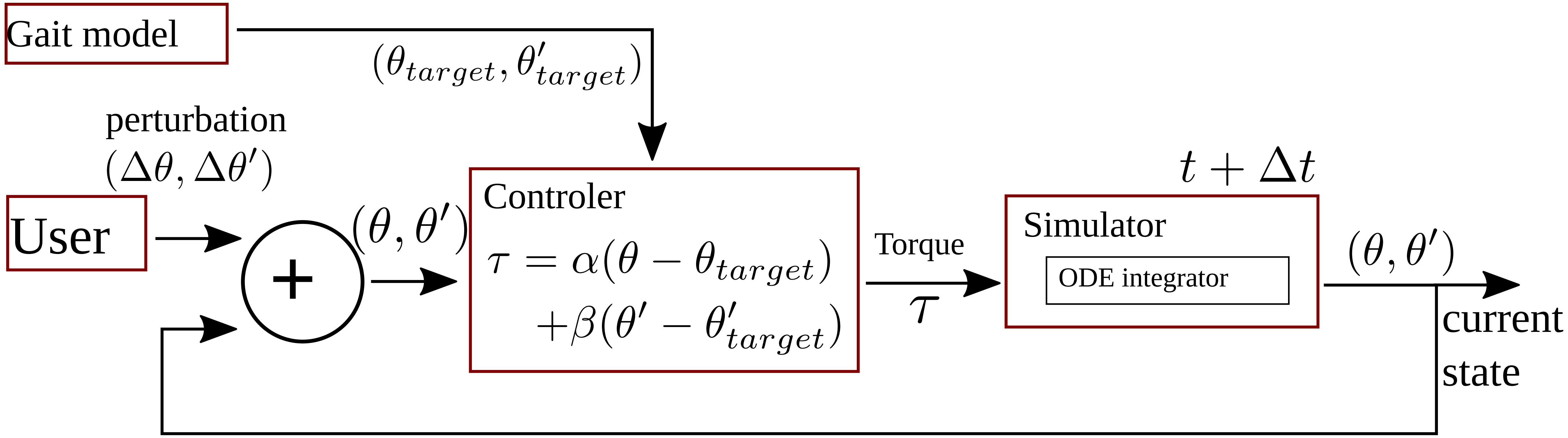
4 - Integrate torque using rigid body simulator

5 - Iterate





Gait model

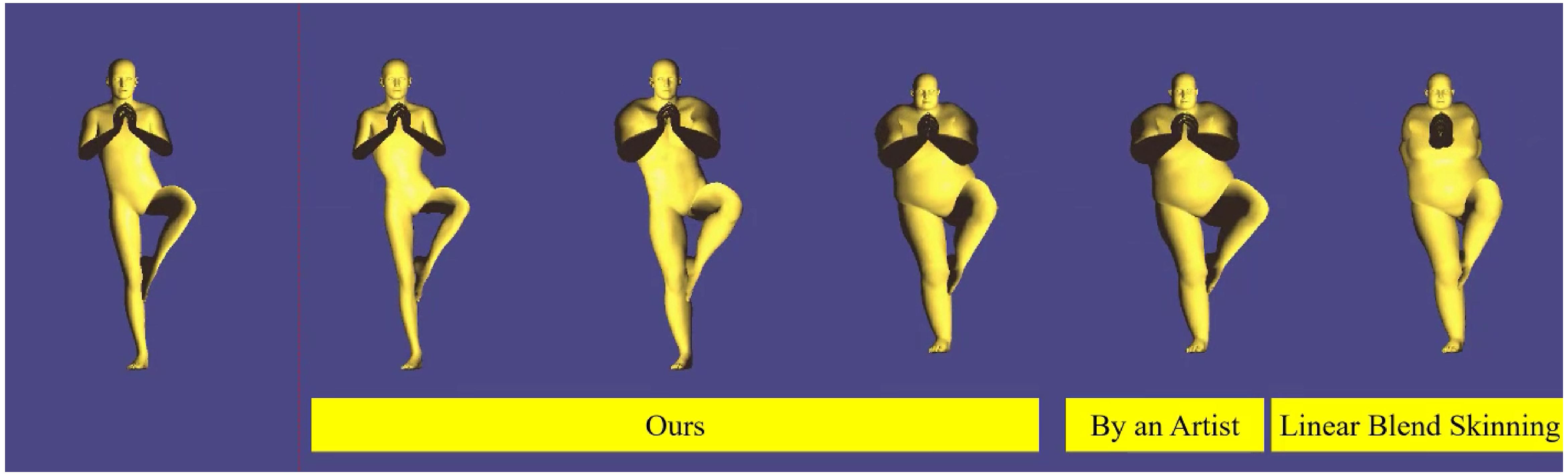


[*M. Raibert and J. Hodgins. Animation of Dynamic Legged Locomotion, ACM SIGGRAPH 2001*]

[*K. Yin et al., SIMBICON: Simple Biped Locomotion Control, ACM SIGGRAPH 2007*]

Motion transfert

- Local coordinates mapping from skeletal motions
  - [C. Hecker et al., Real-time Motion Retargeting to Highly Varied User-Created Morphologies, SIGGRAPH 2008] (Spore)
- Including shape morphology
  - [Z. Liu et al., Surface based Motion Retargeting by Preserving Spatial Relationship, MIG 2018]



Animation design

Based on the *Line of Action*

- [Guay et al., The Line of Action: an Intuitive Interface for Expressive Character Posing, ACM SIGGRAPH Asia 2013]
- [Guay et al., Space-time sketching of character animation, ACM SIGGRAPH 2015]
- [Choi et al., SketchiMo: Sketch-Based Motion Editing for Articulated Characters, ACM SIGGRAPH 2016]

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Automatic synthesis of skeletal animation

Seminal works

- [Evolving Virtual Creatures. Karl Sims. SIGGRAPH 1994]

- [Automated Learning of Muscle-Actuated Locomotion Through Control Abstraction. Radek Grzeszczuk and Demetri Terzopoulos. SIGGRAPH 1995]

- *Optimization toward objective function coupled with rigid bodies simulations*
- *Morphological variation from genetic algorithm*

Optimization of action space

Integrating on complex terrains

- Offline simulation with random terrains
- Learn "reduced action space"
- Motion planning at run time

[Synthesis of Constrained Walking Skills. S. Coros et al. SIGGRAPH 2008]

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Generate as-diverse-as-possible variation of possible motions

[Diverse Motion Variations for Physics-based Character Animation. S. Agrawal et al. SCA 2013.]

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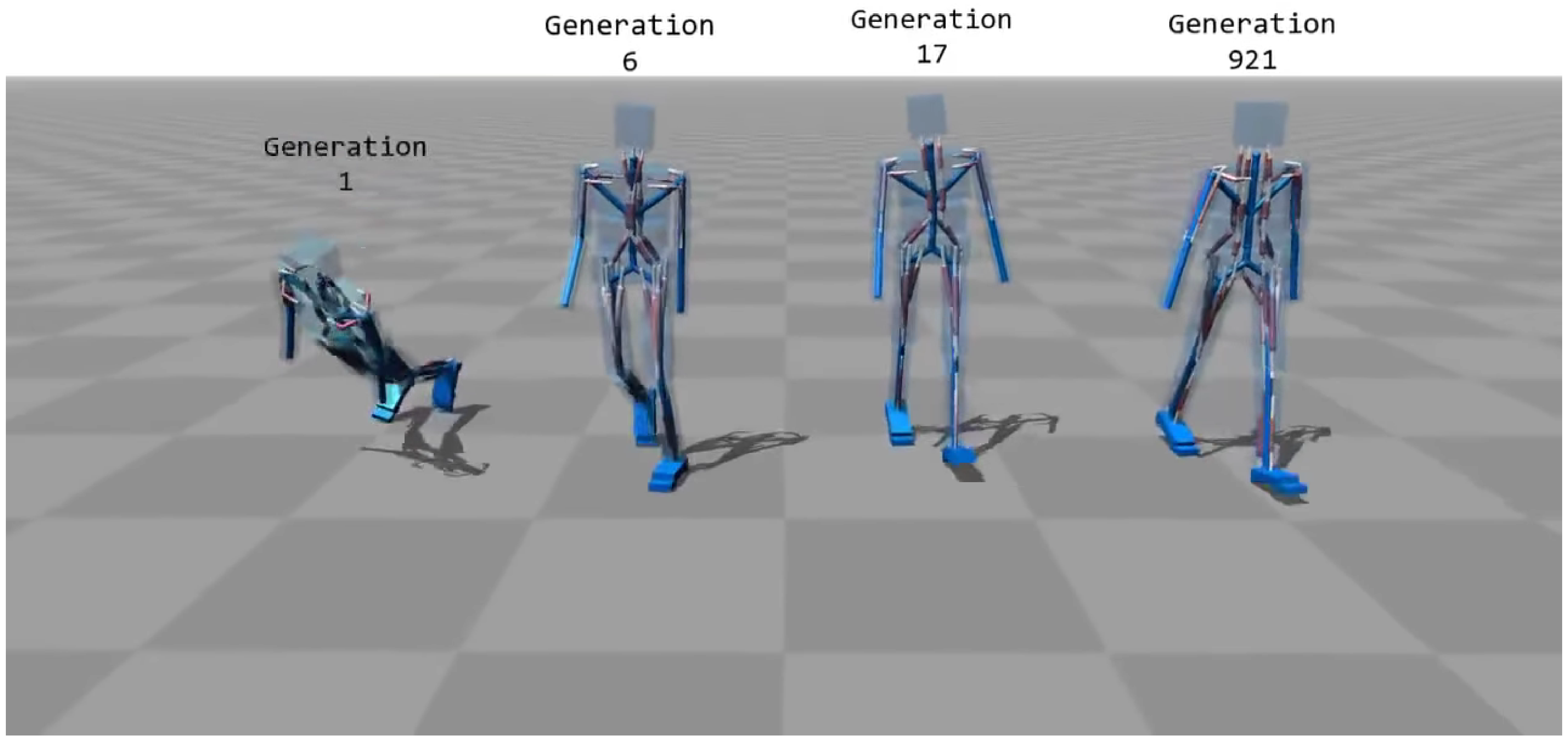
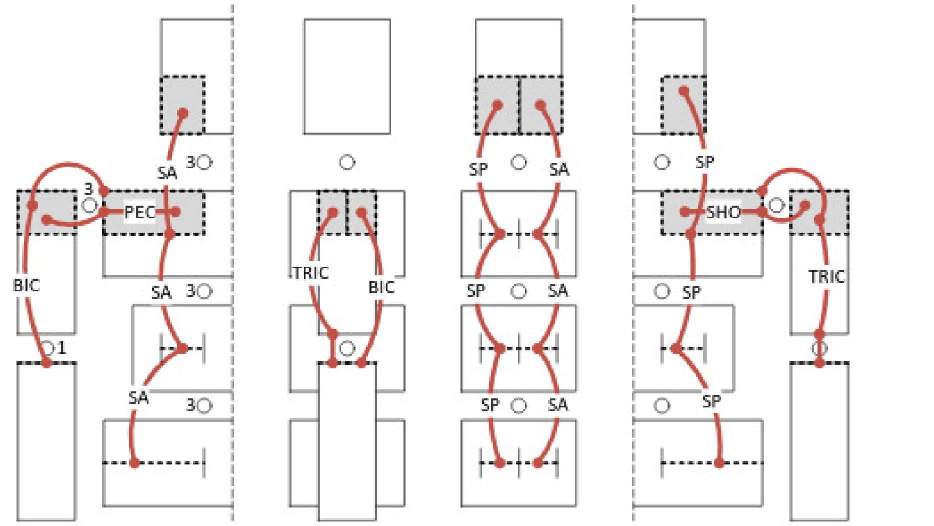
Optimizing muscle activation

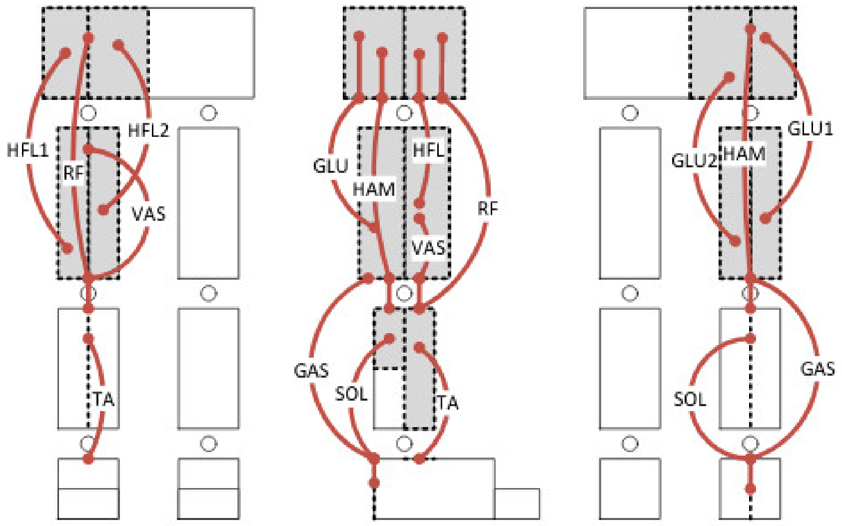
- Take into account simple biomechanical model
- Optimize sequence of activation via reinforcement learning

[Flexible Muscle-Based Locomotion for Bipedal Creatures, T. Geijtenbeek et al. SIGGRAPH Asia 2013.]

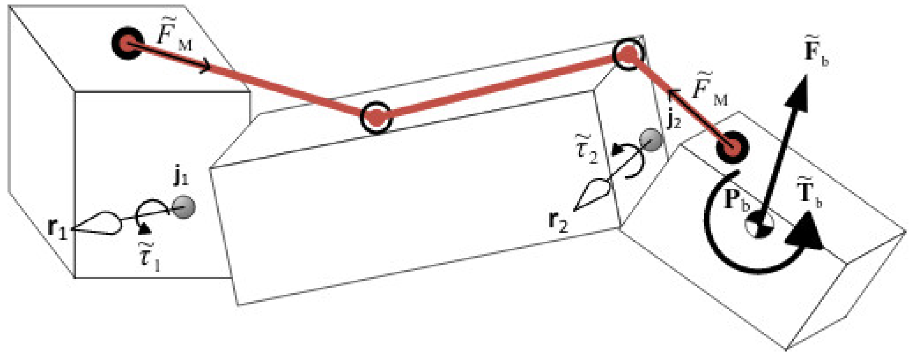




(a) Humanoid upper-body model: front, side arm, side body, and back.



(c) Humanoid lower-body model: front, side and back.

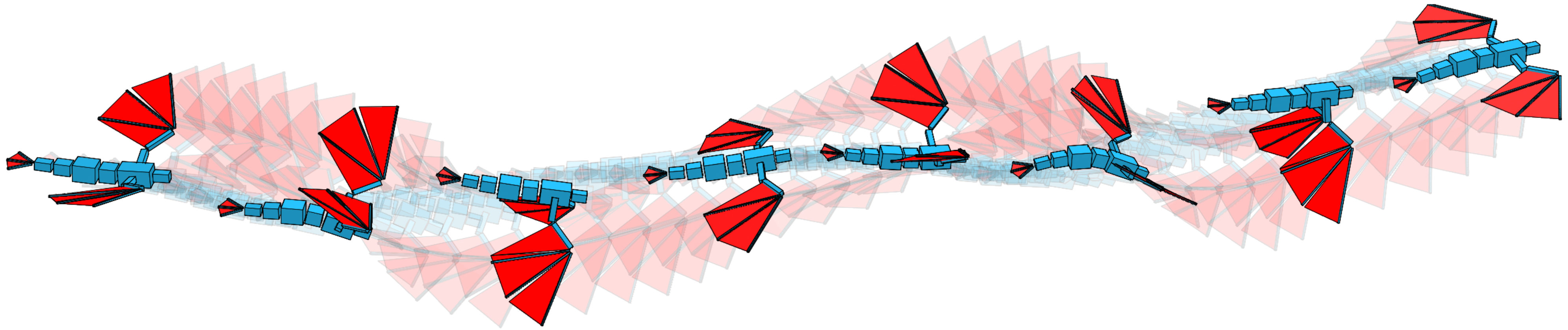


Use of deep learning

Deep reinforcement learning for complex optimization

Learn muscle activation

[Won et al. How to Train Your Dragon: Example-Guided Control of Flapping Flight, ACM SIGGRAPH Asia 2017]



Deep learning for real-time motion control

Learn phase of the motion cycle.

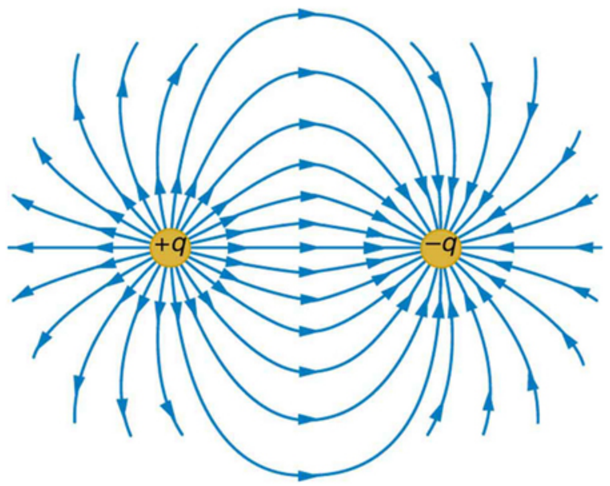
Use large data base of motion capture data

[Holden et al., Phase-Functioned Neural Networks for Character Control, ACM TOG 2017]

Animating crowds of characters

Interaction between particles

Interaction as **force field** (interact at distance)



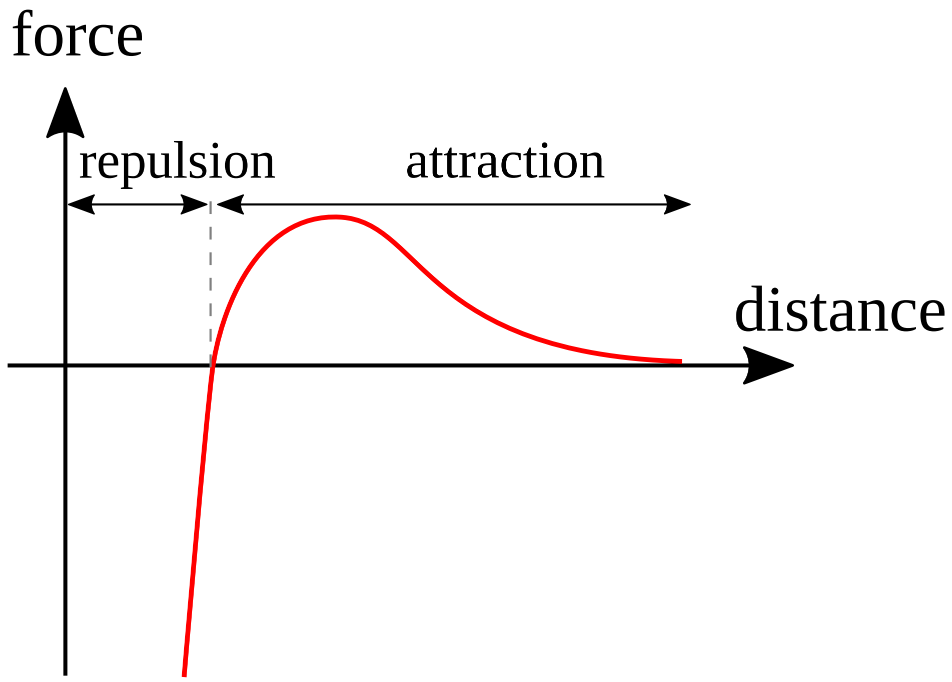
Example of usage

- Models crowd of life-like characters at large scale

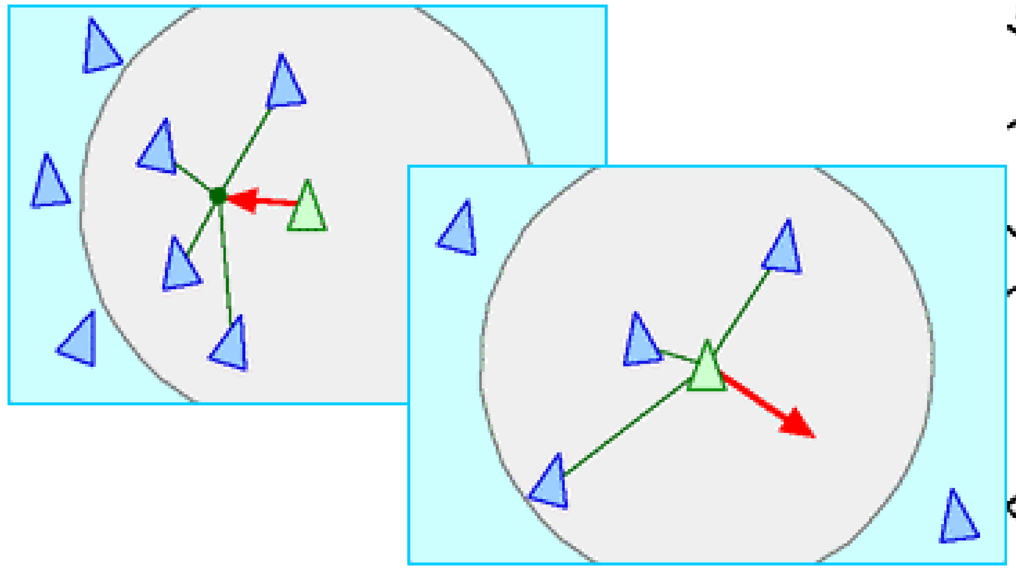
  Inspired from physics particles forces (ex. Lennard-Jones potential)

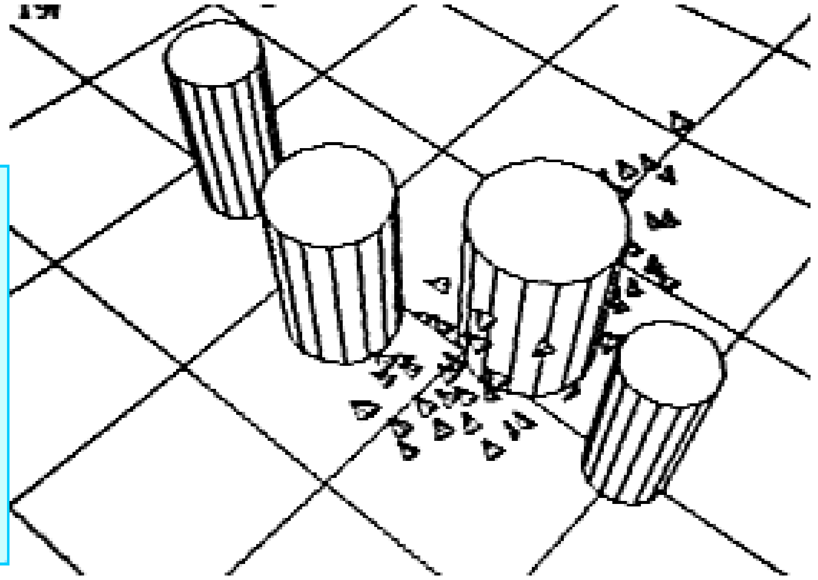
  Attraction at *long-range*

  Repulsion at *short-range*



- First model: **Boids** Craig Reynolds 1987
- Extended later to human crowd modeling





Boids Model

Introduced by

- [*Craig Reynolds. Flocks, Herds, and Schools: A Distributed Behavioral Model, SIGGRAPH 1987*] [link]
- [*Craig Reynolds. Steering Behaviors For Autonomous Characters. Proceedings of Game Developers, 1999*] [link]

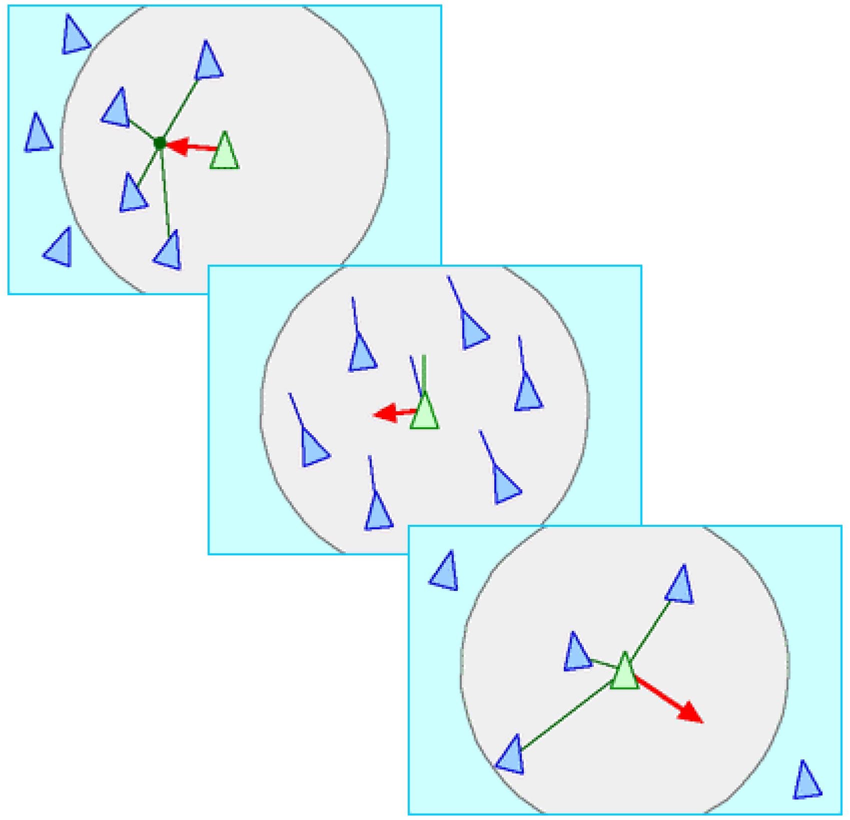
A boid is defined by its

- Position
- Speed
- Forces acting on it

Three basic local *steering behaviors* to model

- **Cohesion** between local particles
- **Alignment** between local particles
- **Separation** between too close particles

=> Leads to emerging global behaviors.



Original video in 1986 (C. Reynolds)

Boids Model - Basic model.

- Set random initial position/speed to N particles.

- Set attraction/repulsion force depending on pairwise distances

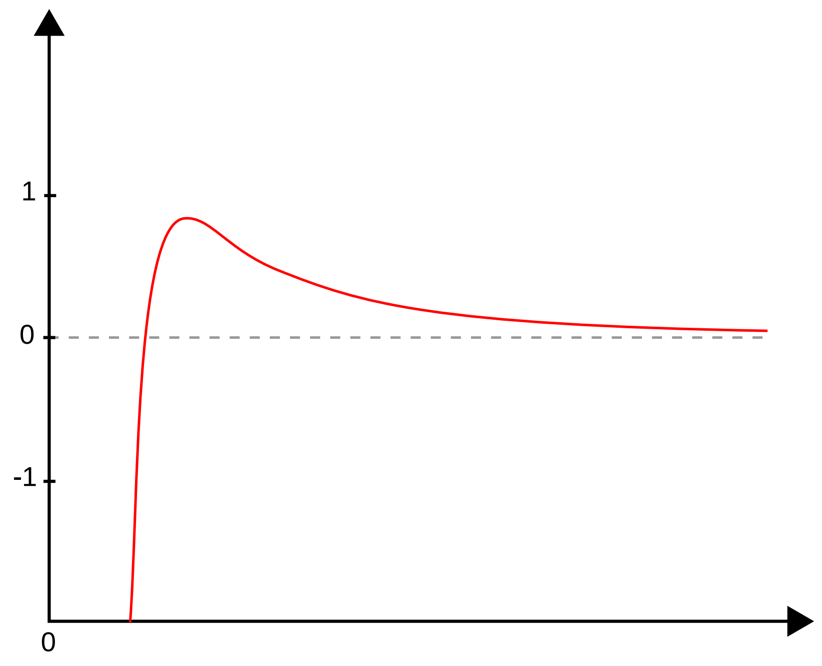
$$F(p_i) = \sum_j f(\|p_j - p_i\|) \frac{p_j - p_i}{\|p_j - p_i\|}$$

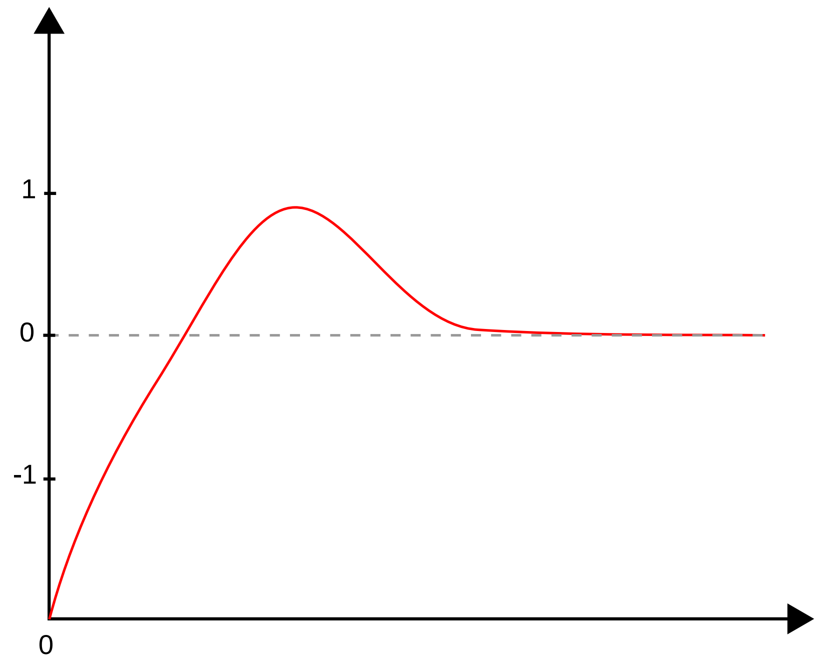
Example

- Inverse of distance $f(x) = \frac{\alpha_1}{x^2} - \frac{\alpha_2}{x^4}$
- Exponential/Gaussian $f(x) = \alpha_1 \exp\left(-k\left(\frac{x - x_0}{x_0}\right)^2\right) - \alpha_2 \exp\left(\frac{x}{x_0}\right)$

- Integrate position and speed through time

$$v^{t+\Delta t}(p_i) = v^t(p_i) + \Delta t \; F(p_i^t)$$
$$p^{t+\Delta t}(p_i) = p^t(p_i) + \Delta t \; v^{t+\Delta t}(p_i)$$





Boids Model - Complexity.

Simple implementation:

```
struct particle { vec3 p, v, f; };
std::vector<particle> boids;
// Initialize N boids ...
// ...



// compute pairwise force
for(int i=0; i<N; ++i)
{
    for(int j=0; j<N; ++j)
    {
        if( i!=j )
        {
            const vec3& pi = boids[i].p;
            const vec3& pj = boids[j].p;
            boids[i].f += force( norm(pi,pj)) / (pi-pj)/norm(pi-pj);
        }
    }
}
// integration
for(int i=0; i<N; ++i)
{
    boids[i].v = boids[i].v + dt * boids[i].f;
    boids[i].p = boids[i].p + dt * boids[i].v;
}
```

- What is the complexity (wrt. N) of this algorithm ?
- Can you think of a way to be more efficient for large N ?

Boids Model - Usage and limitations

- Well adapted to flocks (birds, fishes - looking behavior)
- Display particles using 3D animated model

[redacted]

Additional behaviors

- Objective position/speed value
- Constraints: Obstacle avoidance, limited velocity
- Pursue and evade target/other particles - follow the leader, predators, etc.

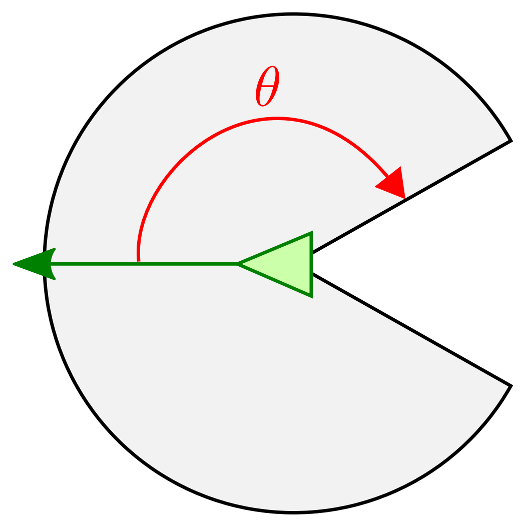
[redacted]

- *Is collision between particles possible ?*
- *Human displacement are mostly guided by vision, what key element is missing in the basic boids force-based model ?*

Boids Model - Extensions

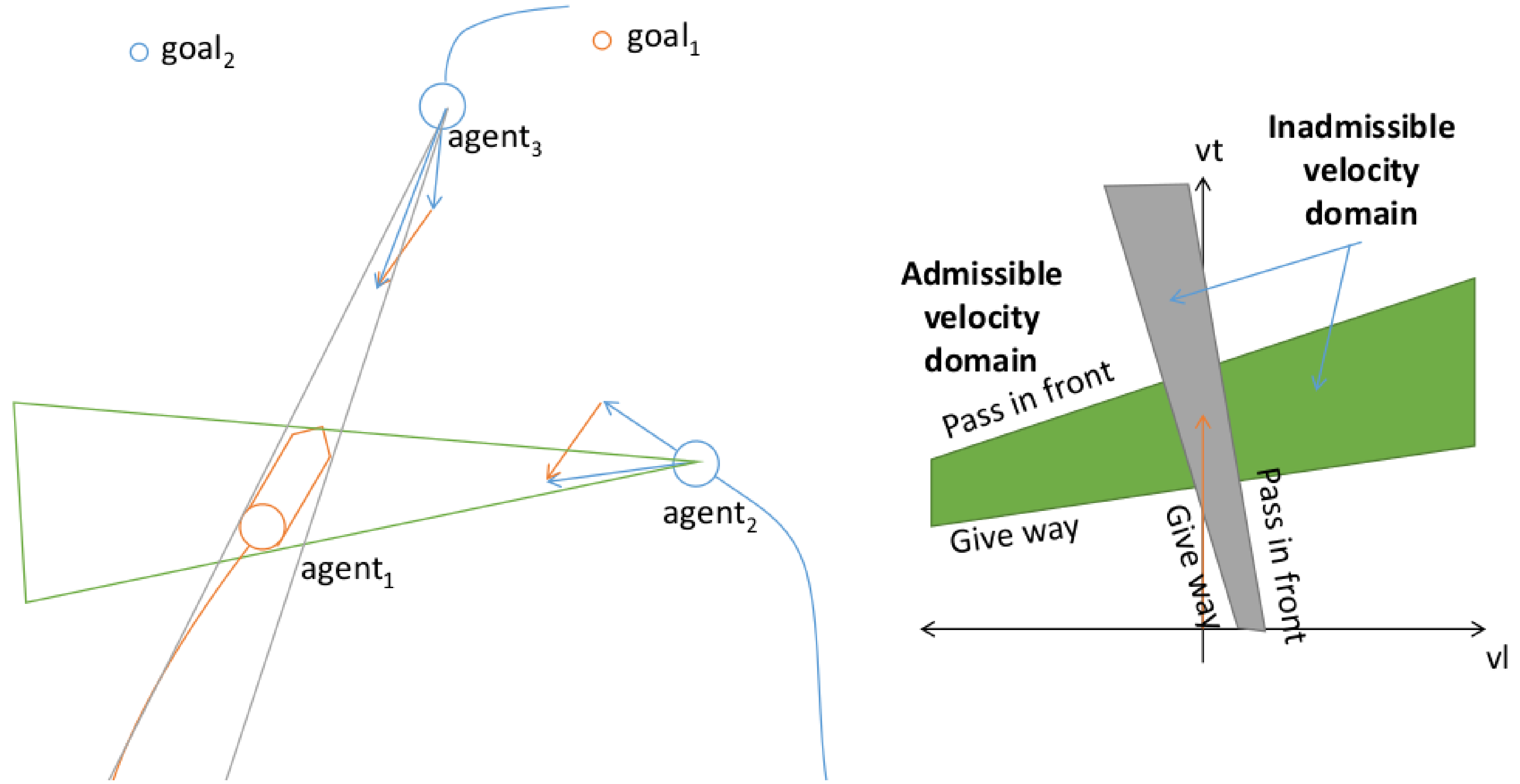
Improving force computation

- Vision-based force computation (angle, speed) : limited view angle



Improving collision avoidance

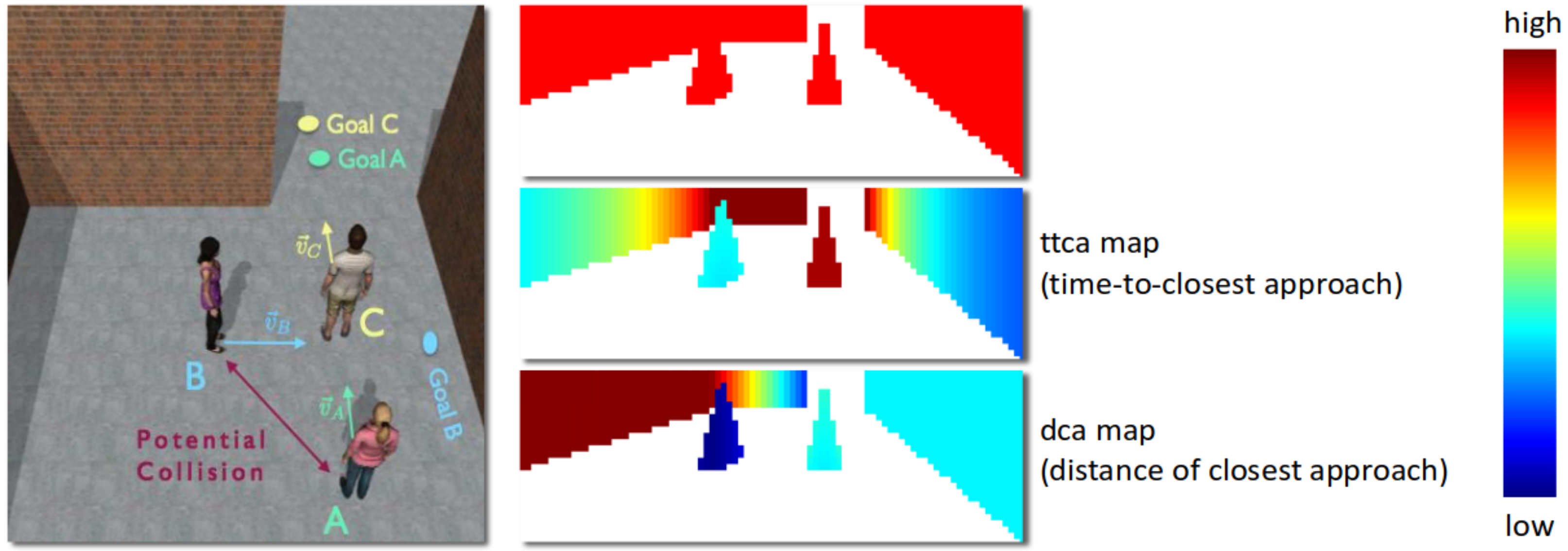
- Velocity-based



[*Experiment-based Modeling, Simulation and Validation of Interactions between Virtual Walkers. J. Pettré et al. SCA 2009*]

Vision based displacement

- Render vision for each individual agent
  - Compute optical flow u.
  - Obstacles are approaching when $\mathrm{div}(u) > 0$
- Best results, but high computational cost.



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[A Synthetic-Vision Based Steering Approach for Crowd Simulation. J. Ondrej et al. SIGGRAPH 2010]
[Character navigation in dynamic environments based on optical flow. A. Lopez et al. EG 2019]

Example of crowd simulation software

Golaem

Small company in Rennes