Shape deformation (II)

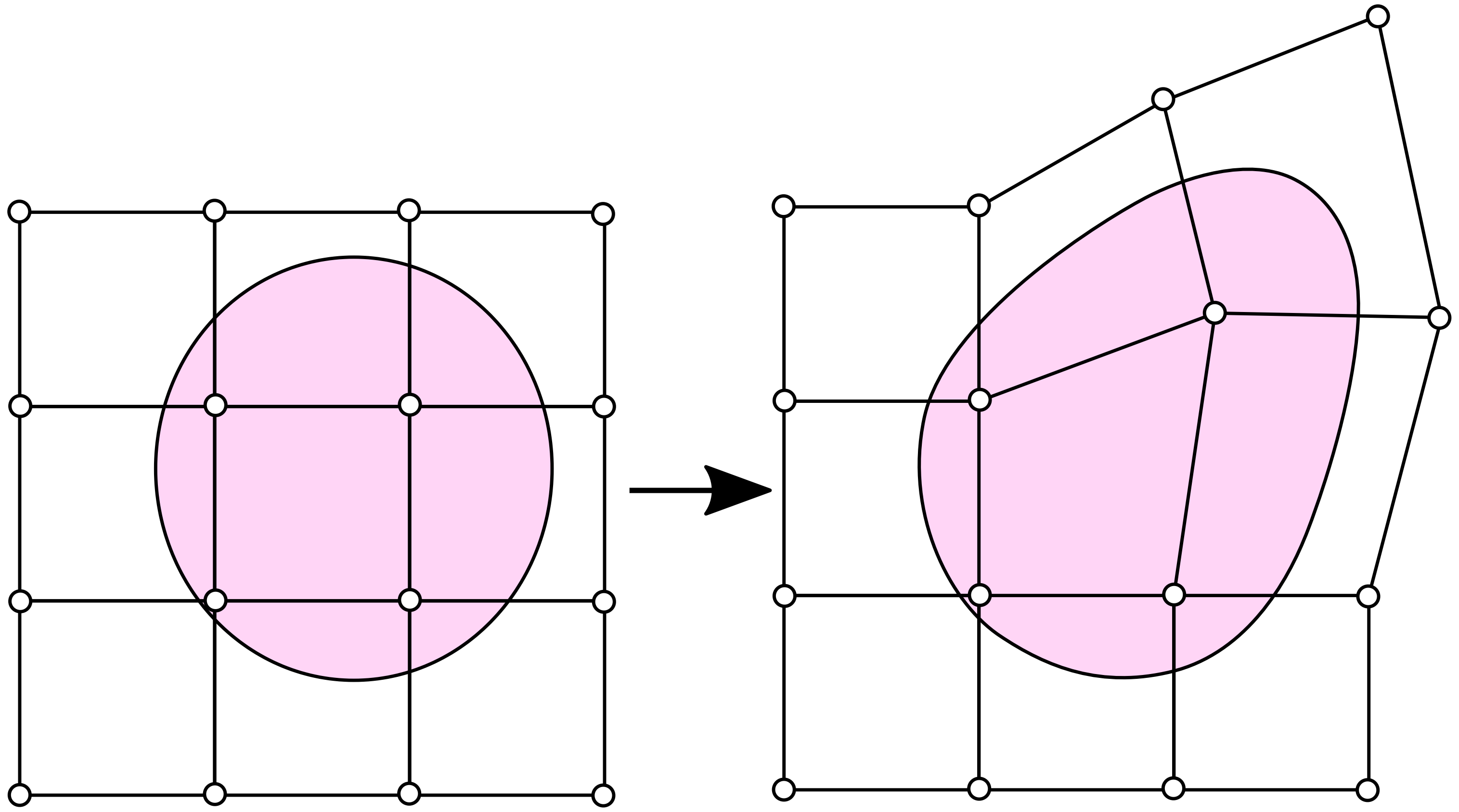
Volume-based deformation

Free Form Deformation
Cages
Vector field

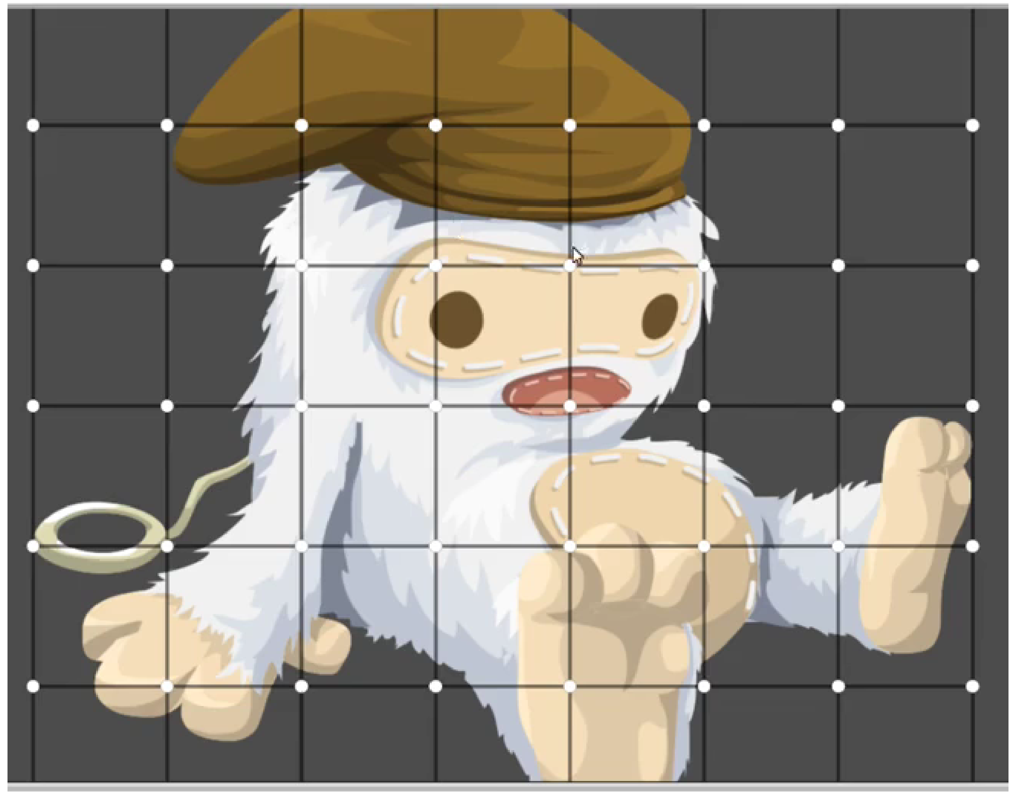
Free Form Deformation

FFD / Grid Based Deformation

- Embeed shape within a rectangular grid (lattice)
- Deform the grid $\rightarrow$ space deformation map



How to compute the space deformation ?



Kestrel Moon

Idea: Approximation function

Bezier, BSpline, NURBS, etc.

Reminder for 1D curve c approximating points $(p_i)_{i=0..N}$ (control polygon)

$$c(u) = \sum_{i=0}^{N} b_i(u)\, p_i$$

$$b_i(u) = \binom{N}{i} u^i\, (1-u)^{N-i}\ , \quad u \in [0,1]$$ (Bernstein Polynomial)

For a 2D surface S approximating points p_{ij}

$$S(u,v) = \sum_i \sum_j b_i(u)\, b_j(v)\ p_{ij}$$ (tensor-product)

For a 3D volume V approximating points on the lattice p_{ijk}

$$V(u,v,w) = \sum_i \sum_j \sum_k b_i(u) b_j(v) b_k(w)\ p_{ijk}$$

$\Rightarrow$ Use V as a spatial deformation on vertex coordinates (u,v,w), use p_{ijk} as grid.

Free Form Deformation

Given

- g_{ijk}: position of the grid coordinates

- $p_m = (x_m, y_m, z_m) \in [0,1]^3$: Initial vertex coordinates (expressed with respect to the grid)

$$\Rightarrow q_m = \sum_i \sum_j \sum_k b_i(x_m)\, b_j(y_m)\, b_k(z_m)\; g_{ijk}$$

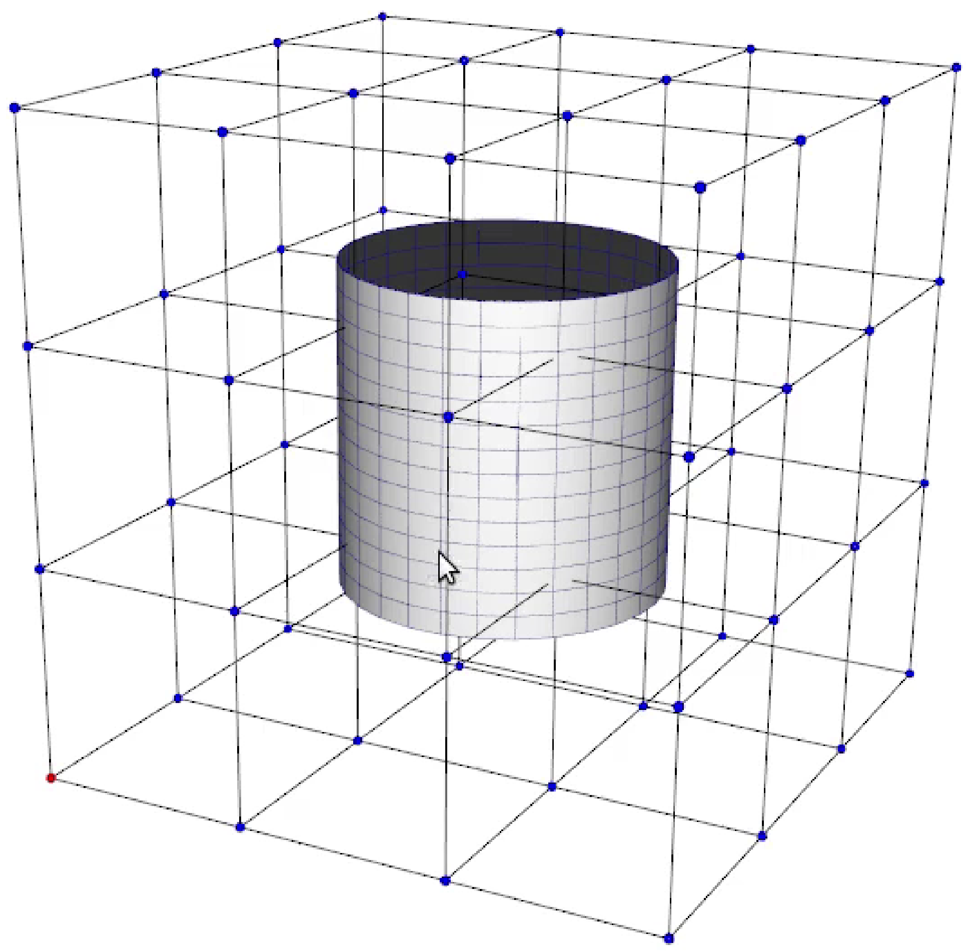
Note:

$$w_{m\,ijk} = b_i(x_m) b_j(y_m) b_k(z_m)$$

is a scalar weights independant of the deformation

$\rightarrow$ can be precomputed

$$\Rightarrow q_m = \sum_{ijk} w_{m\,ijk}\; g_{ijk}$$



FFD limits and extensions

FFD introduced in 1986

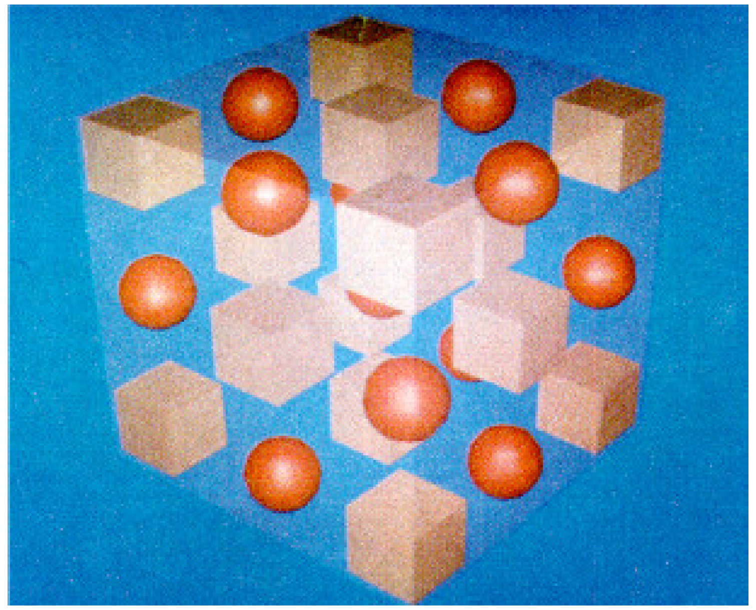
Thomas Sederberg, Scott Parry. Free-Form deformation of solid geometric models, SIGGRAPH 1986.

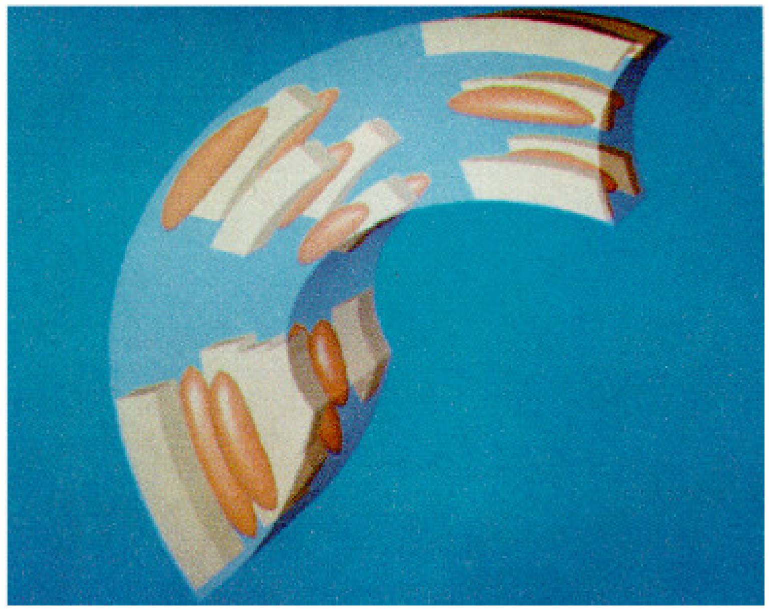
Initially described with Bezier polynomials

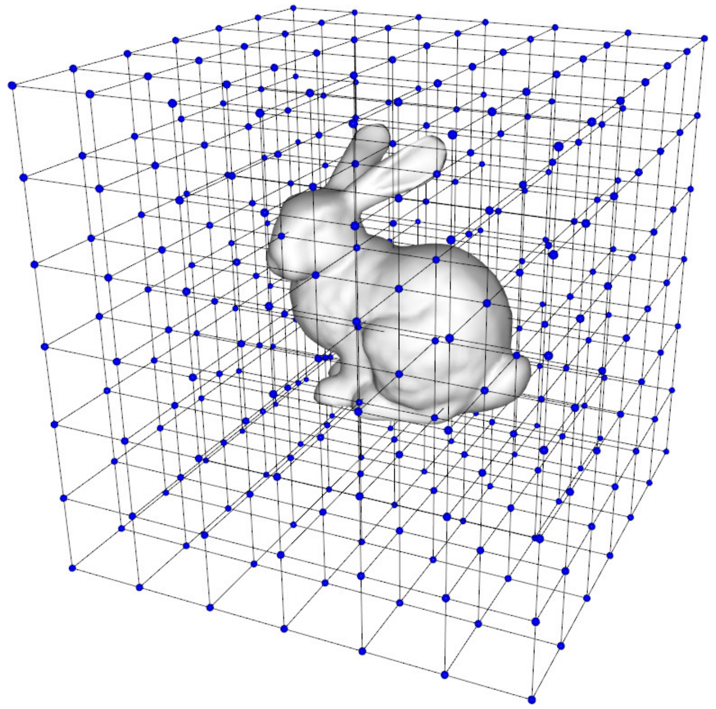
- But extends to Spline with any basis function
- *BSpline, NURBS, etc*

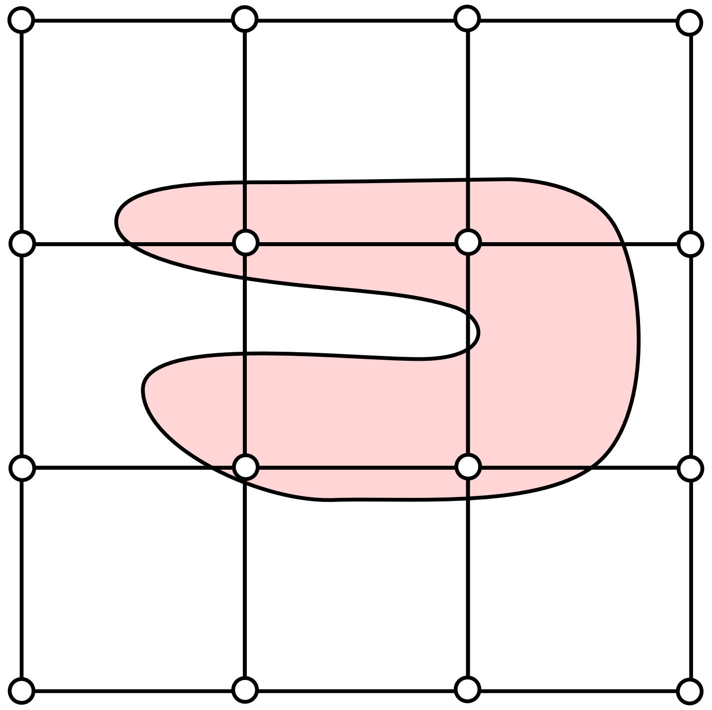
Limitations

- Large grid is hard to manipulate
- Doesn't take into account shape morphology





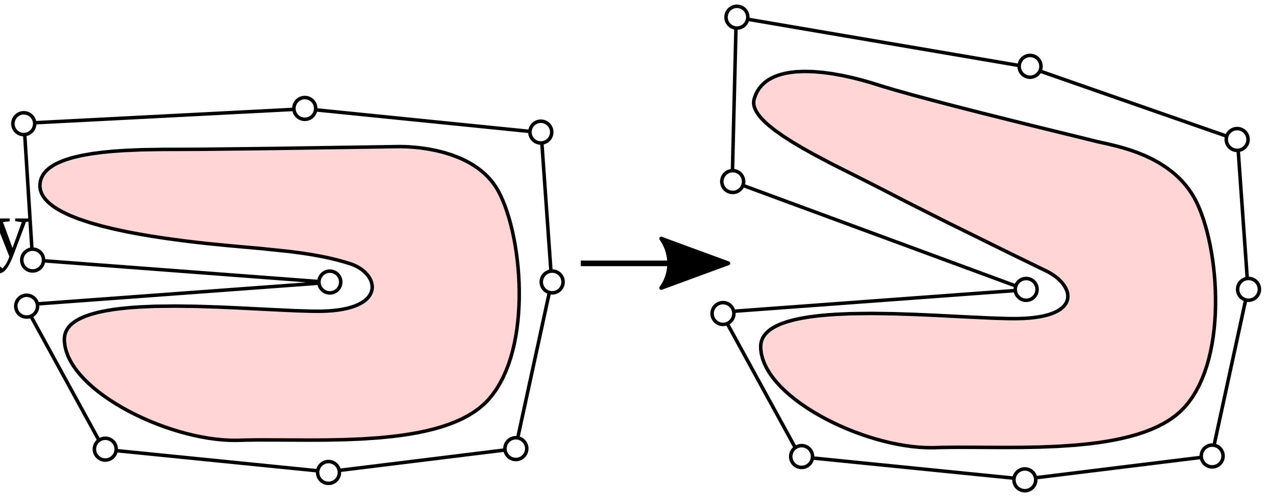




Cage-based Deformation

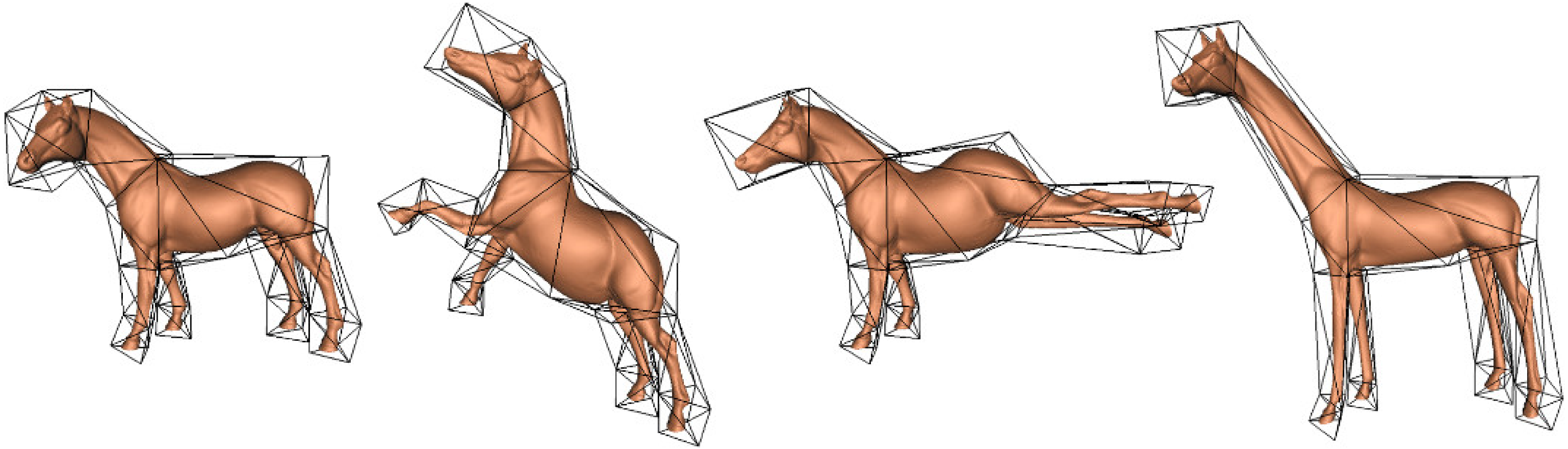
Use a cage surrounding the shape

(+) Adapts deformation locality to shape morphology/topology



Introduced for 3D meshes in 2005

Tao Ju, Scott Schaefer, Joe Warren. Mean value coordinates for closed triangular meshes. SIGGRAPH 2005



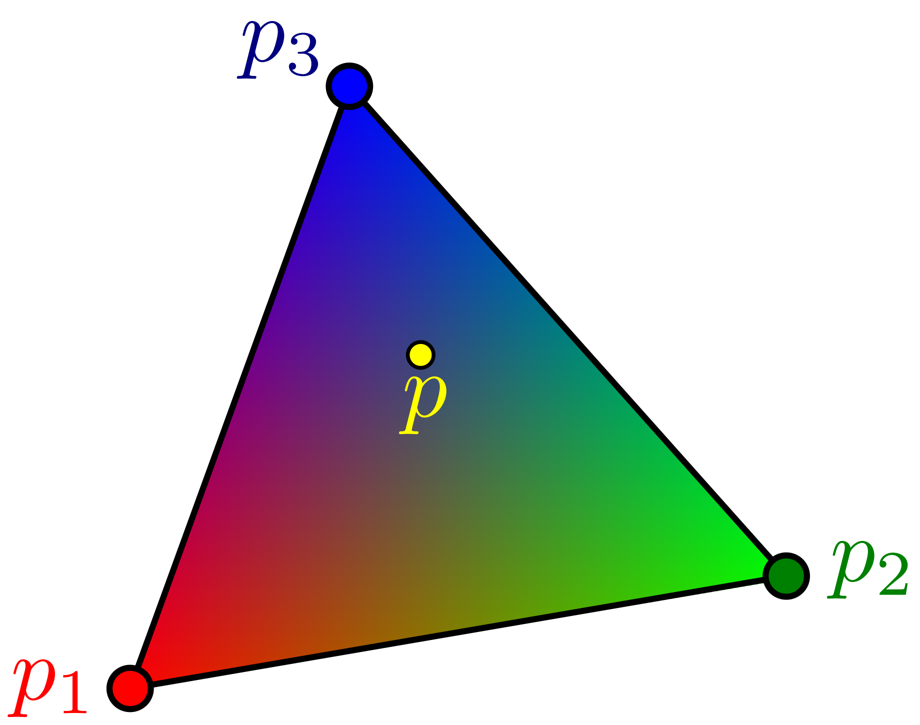
How to compute the spatial deformation from arbitrary cage ?

Barycentric coordinates

Reminder barycentric coordinates for Triangle

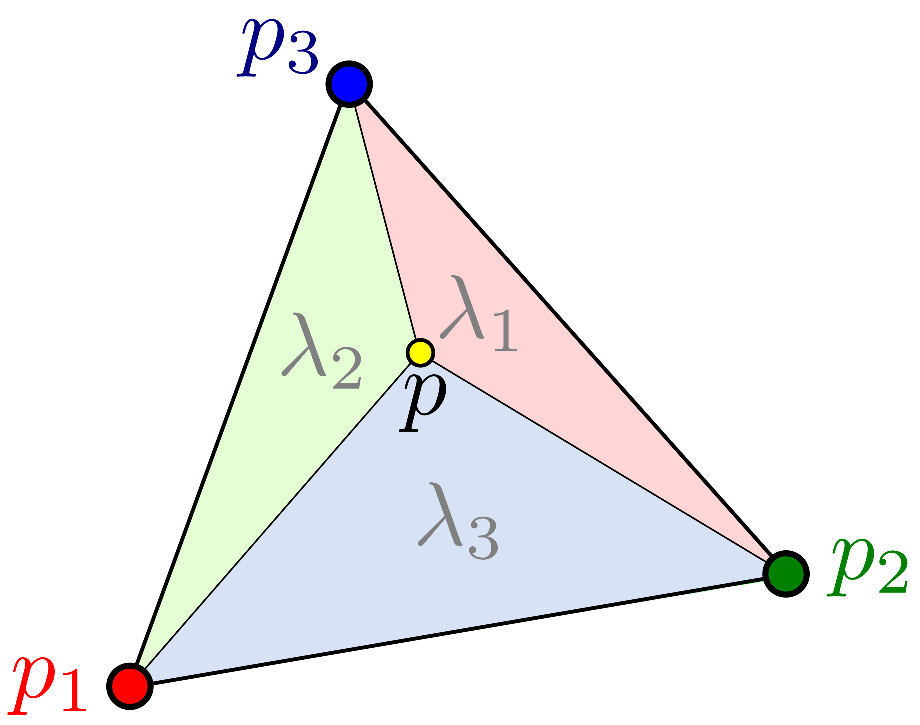
Triangle $(p_1\, p_2\, p_3)$ - barycentric coordinates $(\lambda_1, \lambda_2, \lambda_3)$

$$p \in (p_1\, p_2\, p_3) \Rightarrow p = \lambda_1\, p_1 + \lambda_2\, p_2 + \lambda_3\, p_3$$
$$\lambda_1 + \lambda_2 + \lambda_3 = 1\,, \quad (\lambda_1, \lambda_2, \lambda_3) \in [0,1]^3$$



$$\lambda_1 = area(p_3\ p\ p_2)/\mathcal{A}$$
$$\lambda_2 = area(p_1\ p\ p_3)/\mathcal{A}$$
$$\lambda_3 = area(p_2\ p\ p_1)/\mathcal{A}$$
$$with,\ \mathcal{A} = area(p_1\ p_2\ p_3)$$
$$and,\ area(A\ B\ C) = \tfrac{1}{2}\|(p_2 - p_1) \times (p_3 - p_1)\|$$

Note. Similar for tetrahedron: $p = \lambda_1\, p_1 + \lambda_2\, p_2 + \lambda_3\, p_3 + \lambda_4\, p_4$



Idea: Generalize barycentric coordinates to arbitrary cages

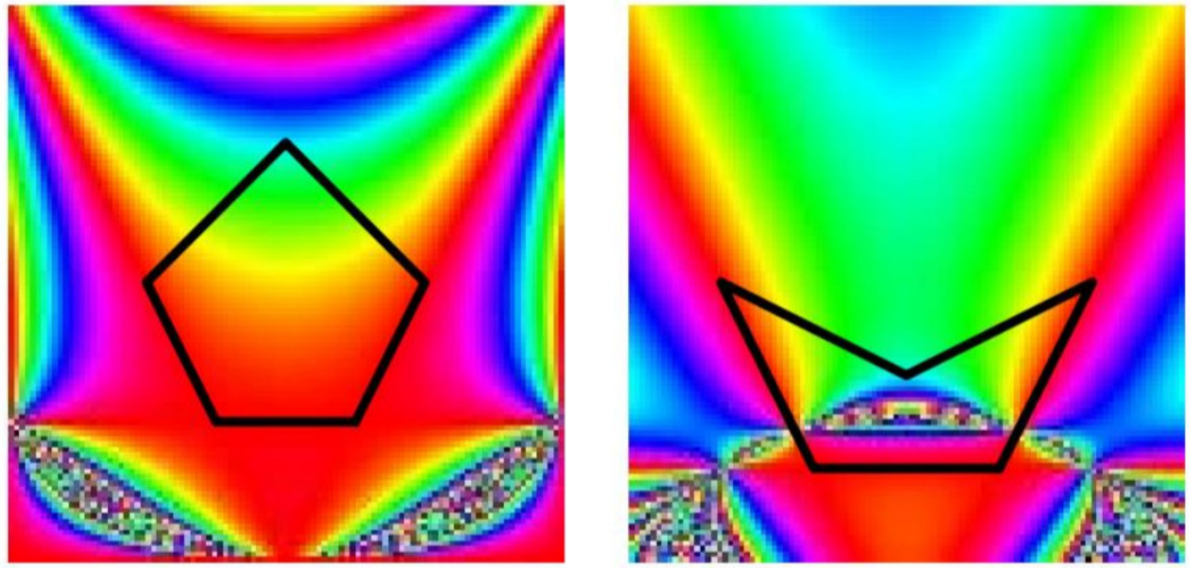
$$p = \sum_i \lambda_i\, p_i = \sum_i \omega_i\, p_i / \sum_i \omega_i$$

Wachspress Coordinates

$$w_i = \frac{A_{i+1} + A_i - B_i}{A_{i+1}\, A_i}$$

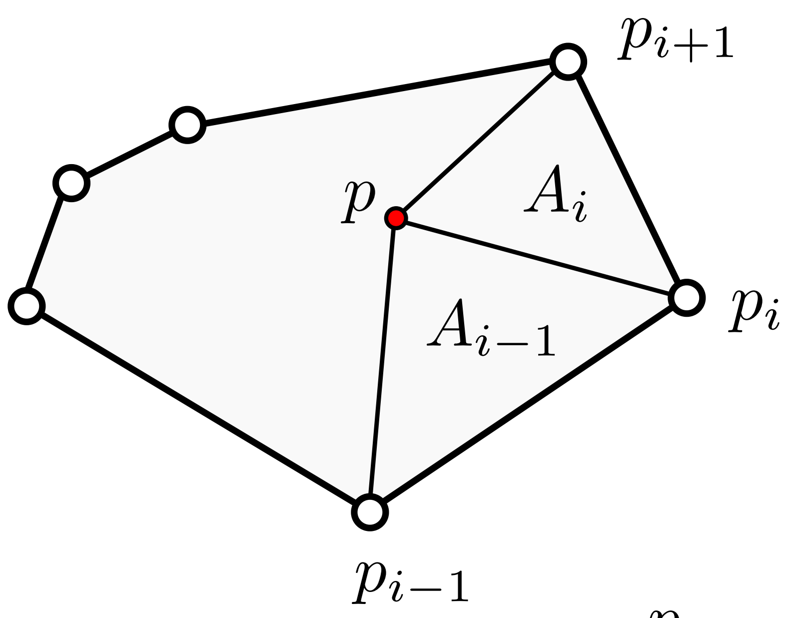
$$w_i = \frac{cot(\alpha_{i-1}) + cot(\beta_i)}{\|p_i - p\|}$$

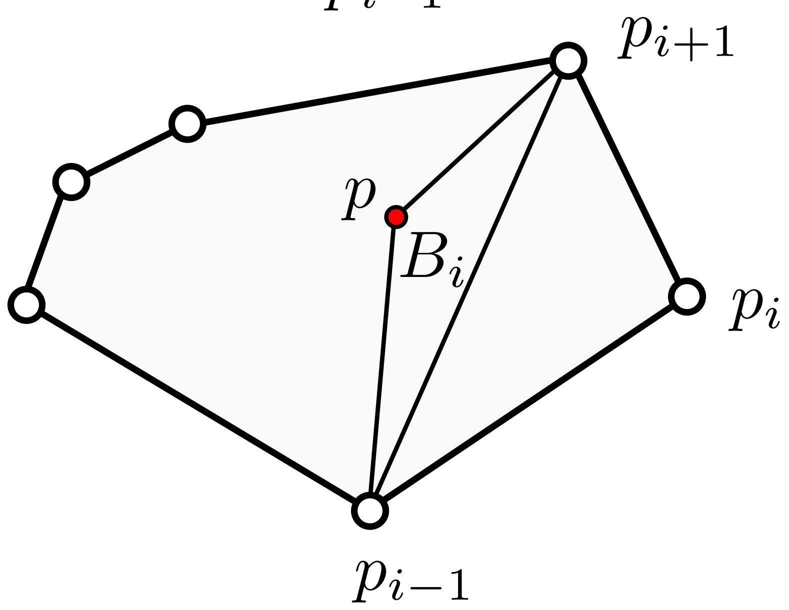
Introduced in 1975, Eugene Wachspress, A Rational Finite Element Basis.

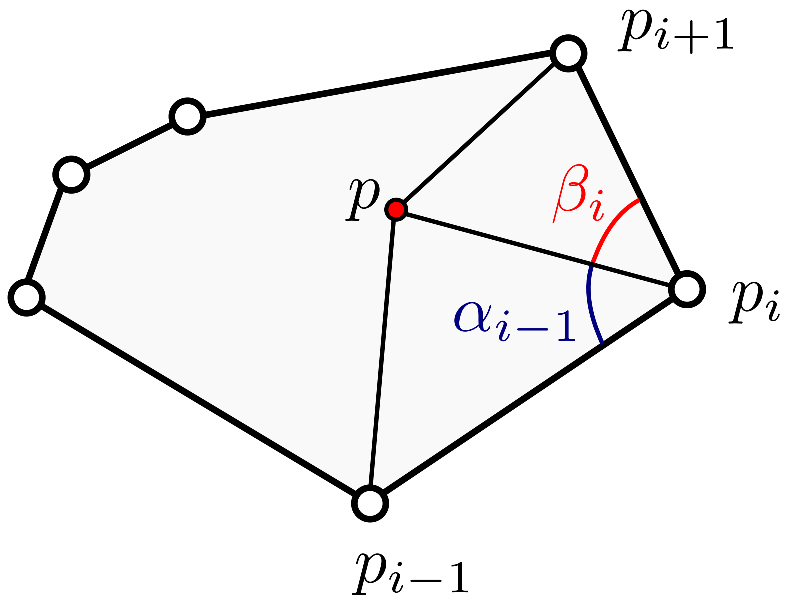


(-) Limited to 2D

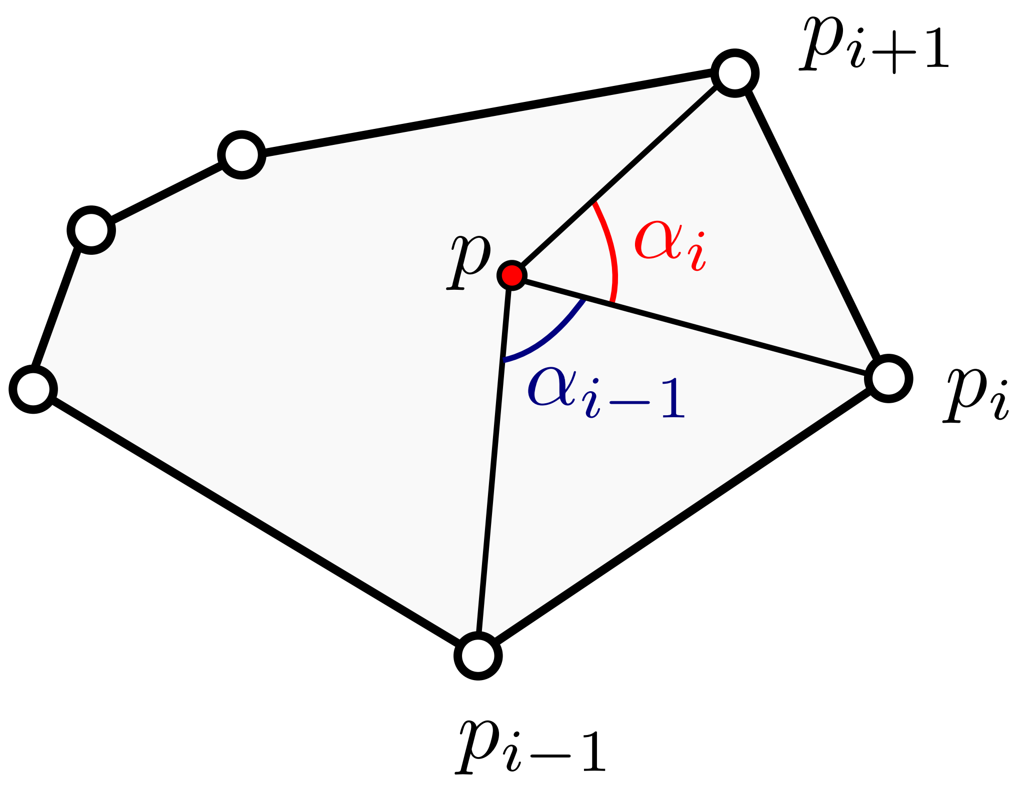
(-) Limited to convex polygons (negative weights, poles)







Mean Value

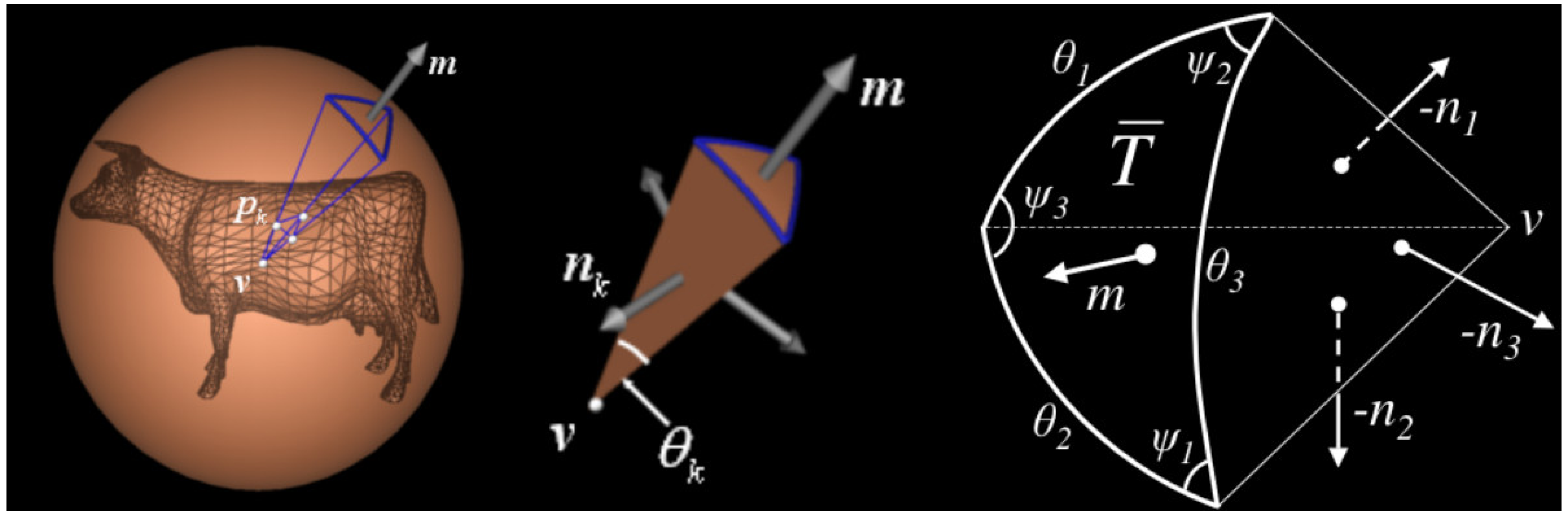


In 2D

$$\omega_i = \frac{\tan(\alpha_{i-1}/2) + \tan(\alpha_i/2)}{\|p_i - p\|}$$

Michael S. Floater. Mean value coordinates. CAGD, 2003

In 3D



$$\omega_i = \frac{n_i \cdot m}{n_i \cdot (p_i - v)}$$

$$m = \frac{1}{2} \sum_i \theta_i \, n_i$$

Michael S. Floater, Géza Kos, Martin Reimers. Mean value coordinates in 3D. CAGD, 2005
Tao Ju, Scott Schaefer, Joe Warren. Mean Value Coordinates for Closed Triangular Meshes. SIGGRAPH 2005

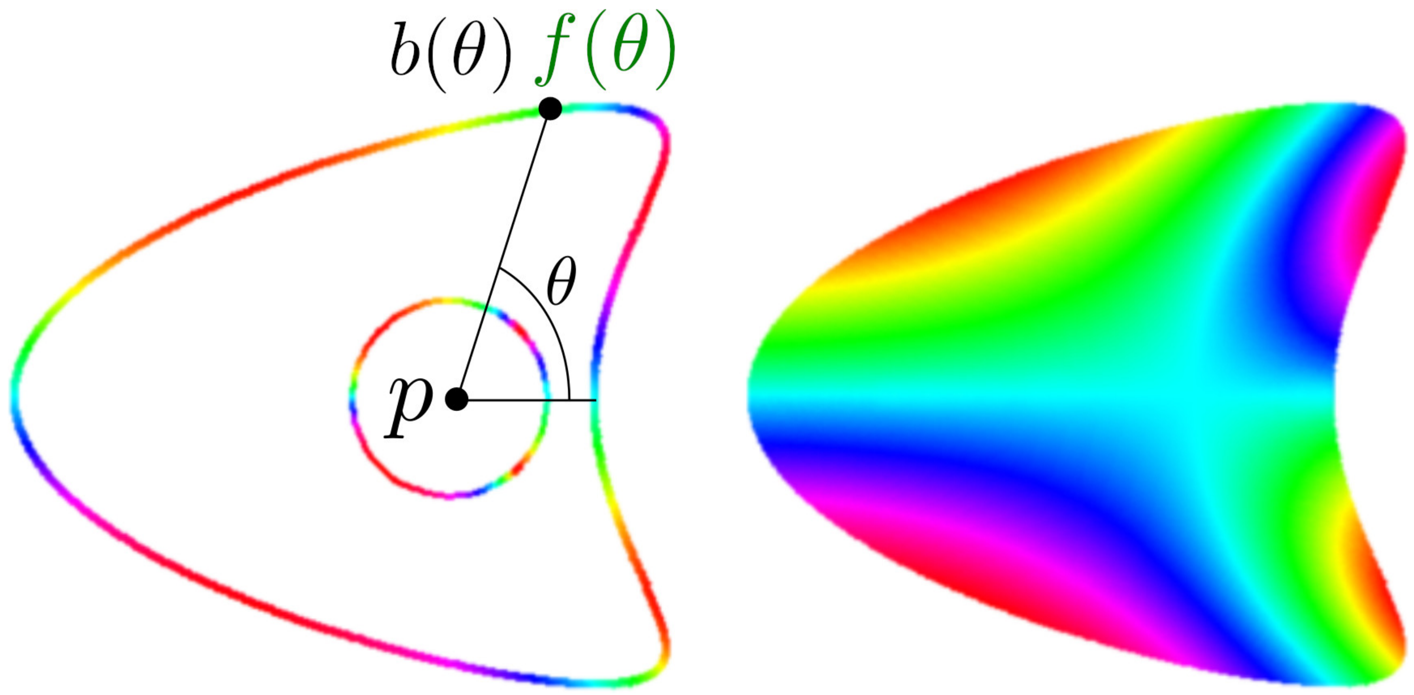
Mean Value - Idea in 2D

Look for an interpolant.

- Given a 2D boundary curve $b(\theta)$, with values $f(\theta)$
- Mean-value associated to point p:

$$\hat{f}(p) = \frac{\displaystyle\int_{\theta=0}^{2\pi} \kappa(\theta)\, f(\theta)\, \mathrm{d}\theta}{\displaystyle\int_{\theta=0}^{2\pi} \kappa(\theta)\, \mathrm{d}\theta}, \quad \kappa(\theta) = 1/\|b(\theta) - p\|$$

Averaged value projected around a unit circle, weighted by the inverse distance.

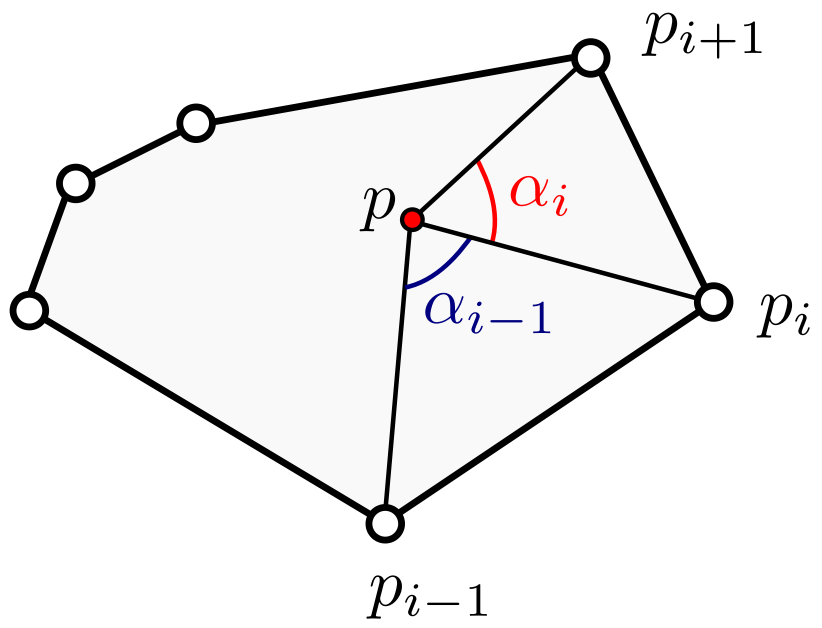


For a piecewise linear boundary

- Over the edge $p_i\, p_{i+1}$

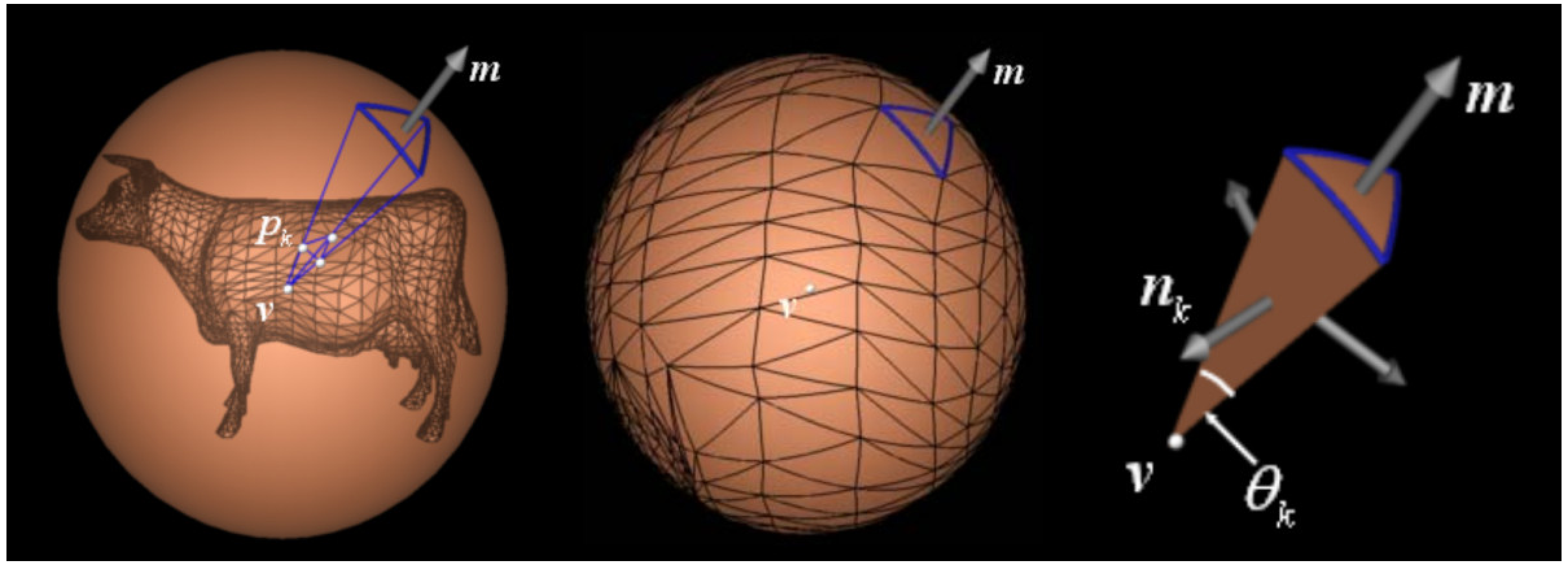
$$\int_{\theta=\theta_1}^{\theta=\theta_2} \kappa(\theta)\, f(\theta) \mathrm{d}\theta = \cdots = \left(\frac{f(p_i)}{\|p_i - p\|} + \frac{f(p_{i+1})}{\|p_{i+1} - p\|} \right) tan \left(\frac{\theta_2 - \theta_1}{2} \right)$$

$$\Rightarrow \omega_i = \frac{\tan(\alpha_{i-1}/2) + \tan(\alpha_i/2)}{\|p_i - p\|}$$



Mean Value - Idea in 3D

In 3D: Integrate over a triangulated sphere

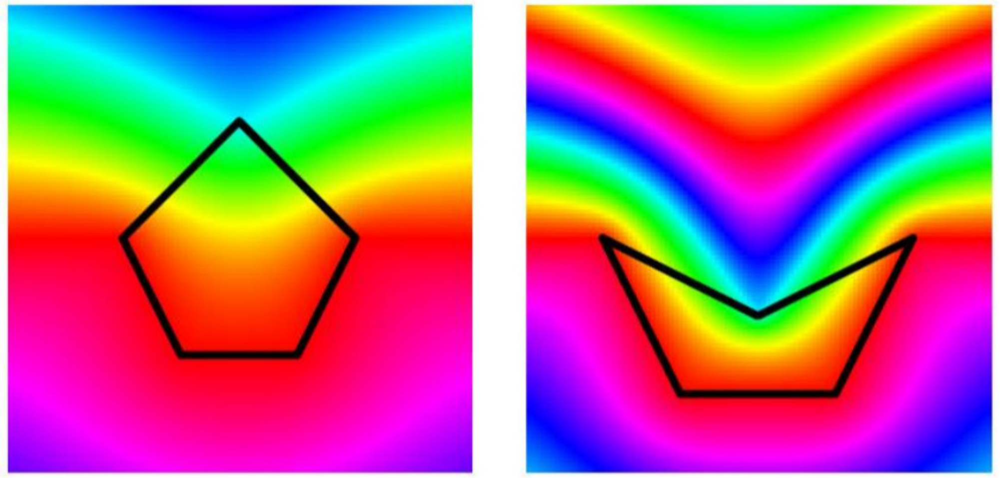


Mean Value Coordinates for Closed Trianular Meshes. T. Ju et

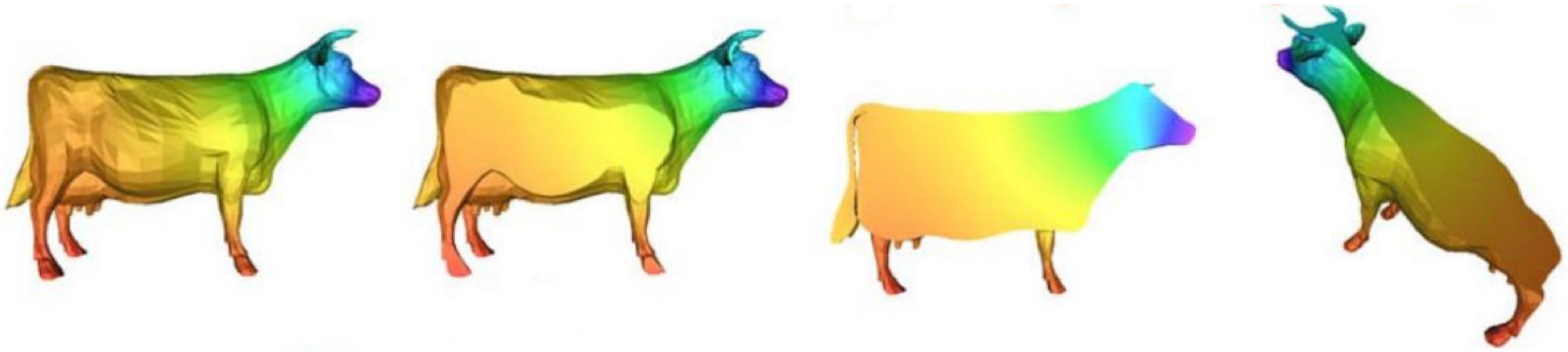
// Robust evaluation on a triangular mesh
for each vertex p_j with values f_j
$\quad d_j \leftarrow \|p_j - x\|$
$\quad$ if $d_j < \varepsilon$ return f_j
$\quad u_j \leftarrow (p_j - x)/d_j$
$totalF \leftarrow 0$
$totalW \leftarrow 0$
for each triangle with vertices p_1, p_2, p_3 and values f_1, f_2, f_3
$\quad l_i \leftarrow \|u_{i+1} - u_{i-1}\|$ $\qquad$ *// for* $i = 1,2,3$
$\quad \theta_i \leftarrow 2\arcsin[l_i/2]$
$\quad h \leftarrow (\sum \theta_i)/2$
$\quad$ if $\pi - h < \varepsilon$
$\qquad$ *// x lies on t, use 2D barycentric coordinates*
$\qquad w_i \leftarrow \sin[\theta_i] d_{i-1} d_{i+1}$
$\qquad$ return $(\sum w_i f_i)/(\sum w_i)$
$\quad c_i \leftarrow (2\sin[h]\sin[h-\theta_i])/(\sin[\theta_{i+1}]\sin[\theta_{i-1}]) - 1$
$\quad s_i \leftarrow \mathrm{sign}[det[u_1, u_2, u_3]]\sqrt{1 - c_i{}^2}$
$\quad$ if $\exists i, |s_i| \leq \varepsilon$
$\qquad$ *// x lies outside t on the same plane, ignore t*
$\qquad$ continue
$\quad w_i \leftarrow (\theta_i - c_{i+1}\theta_{i-1} - c_{i-1}\theta_{i+1})/(d_i \sin[\theta_{i+1}] s_{i-1})$
$\quad totalF += \sum w_i f_i$
$\quad totalW += \sum w_i$
$f_x \leftarrow totalF/totalW$

Mean Value - Results

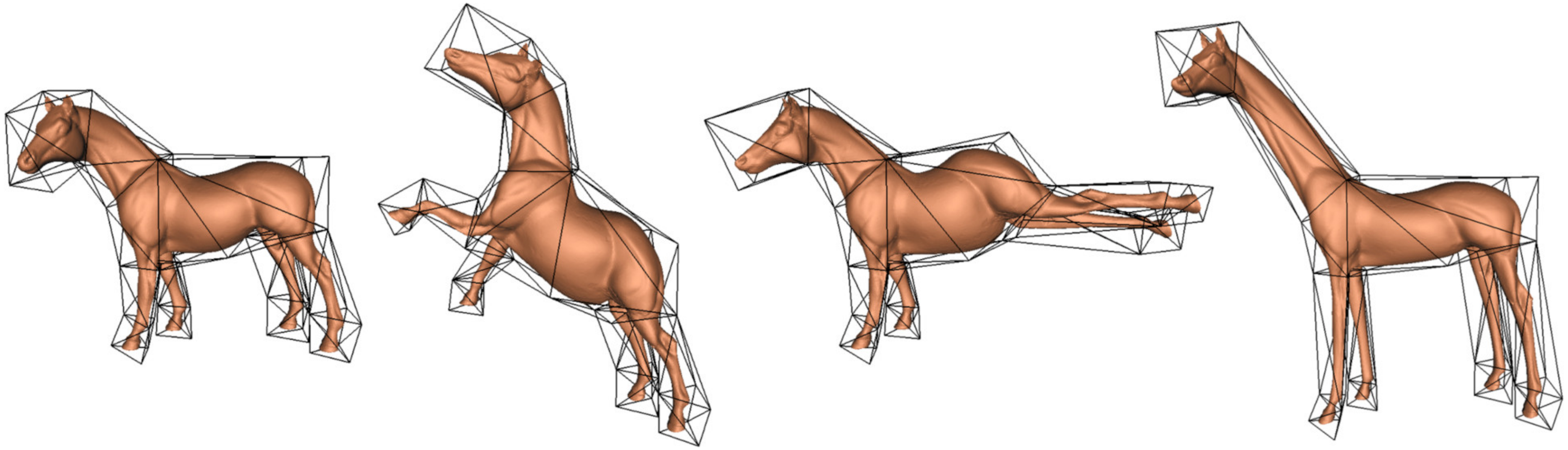
Coordinates in 2D



Coordinates in 3D



Application to shape deformation



Harmonic Coordinates

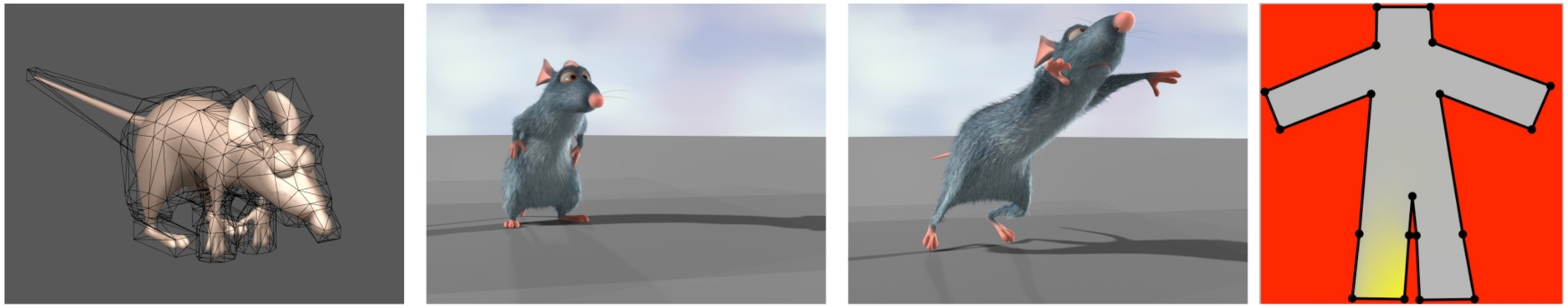
ω solution of the harmonic equation

$$\triangle \omega(p) = 0$$

$$\omega_i(p_j) = \delta_{ij}$$

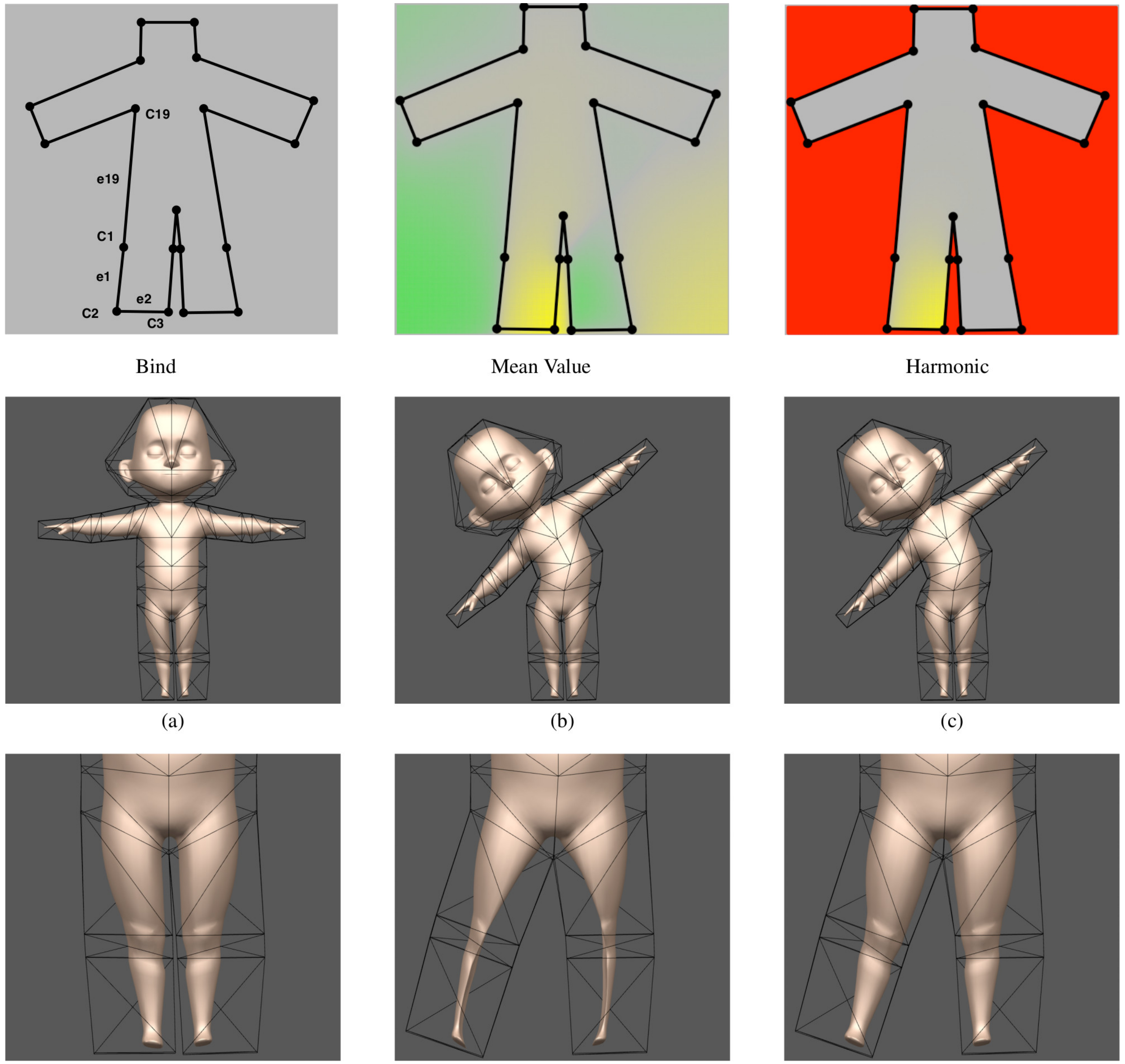
(-) No-closed form solution: requires numerical solver.

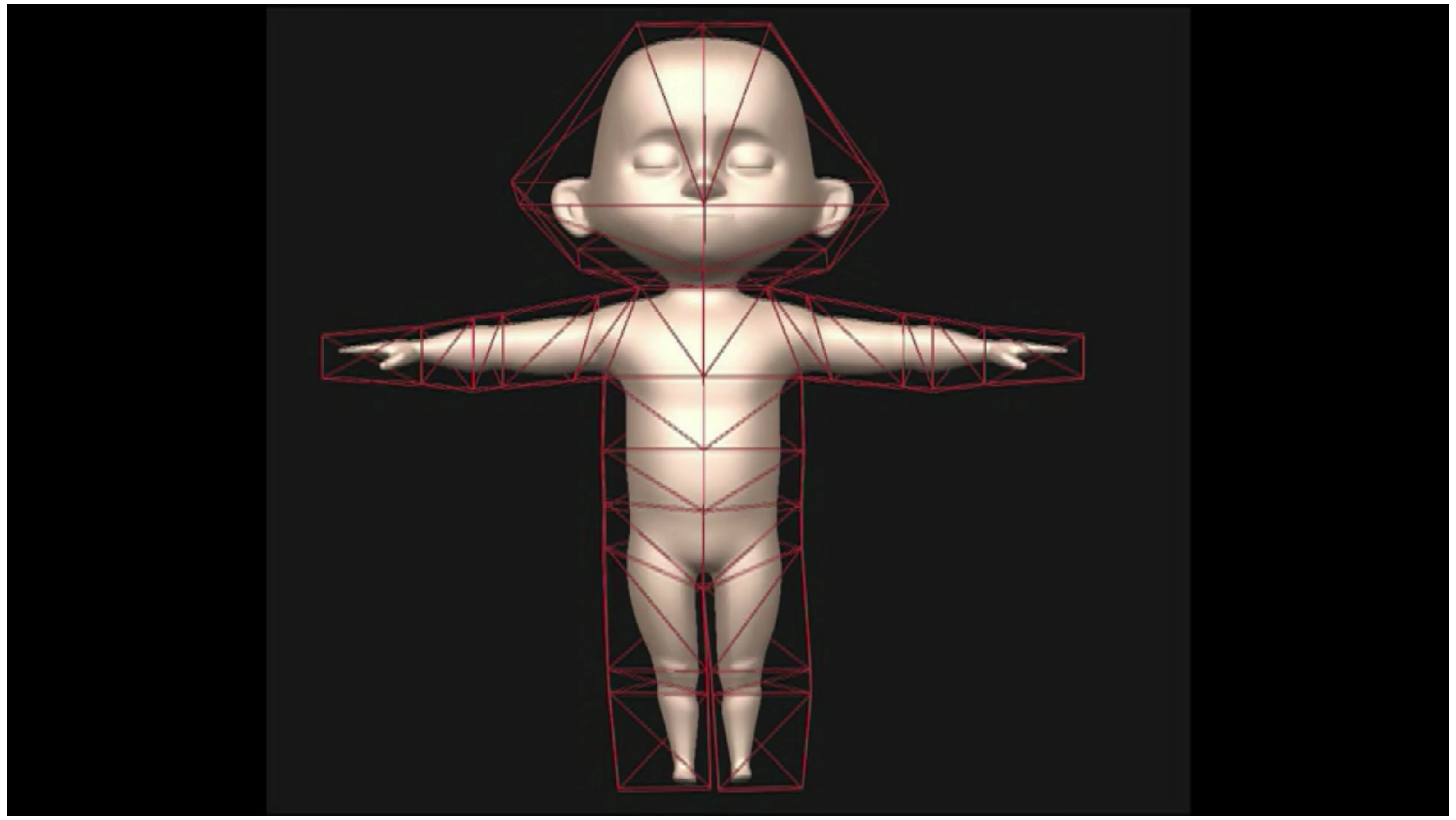
(+) Positive weights, even for concave cages



Pushkar Joshi, Mark Meyer, Tony DeRose. Harmonic Coordinates for Character Articulation. SIGGRAPH 2007.

Harmonic Coordinates VS Mean Value Coordinates



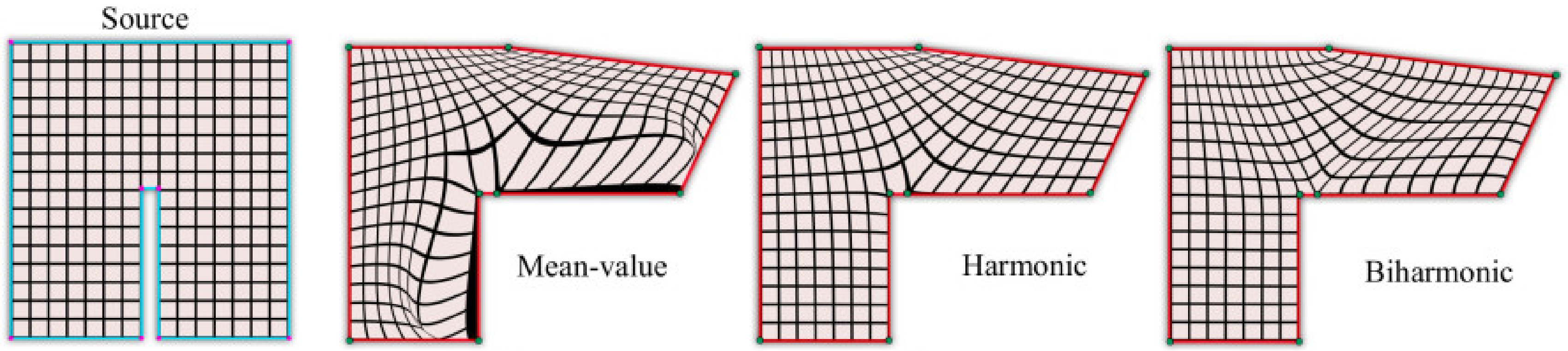


Extensions

Other coordinates

Green coordinates

Green Coordinates. Y. Lipman et al. SIGGRAPH



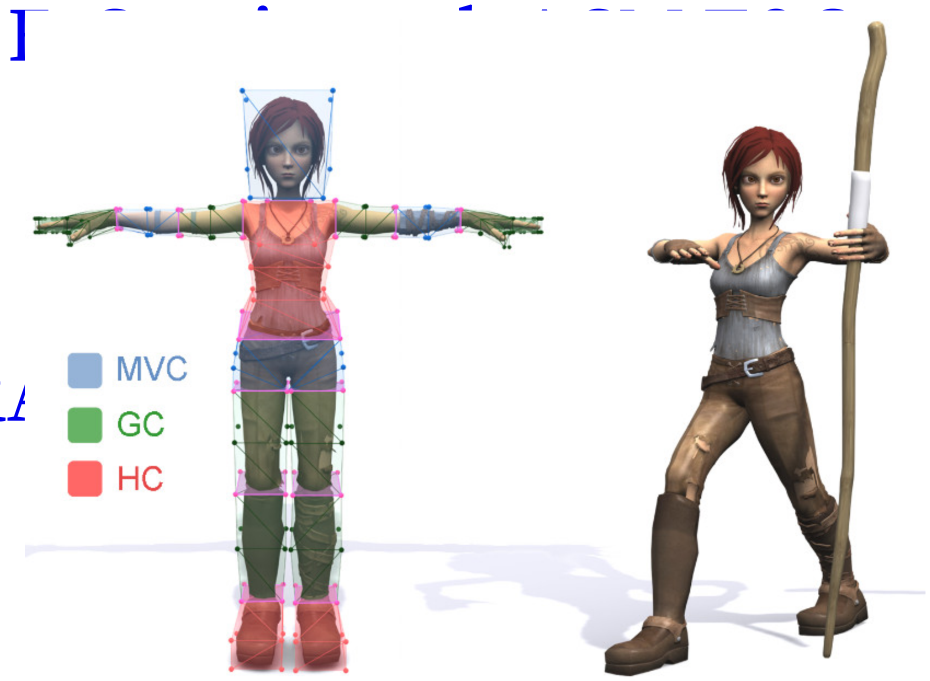
Bi-Harmonic Coordinates

Biharmonic Coordinates. O. Weber et al. CGF 2012

Multi-cage

*Cages: A multilevel, multi-cage-based system for mesh deformation. I

2013



MVC on quad meshes

Mean value coordinates for quad cages in 3D. J.-M. Thiery et al. SIGGR

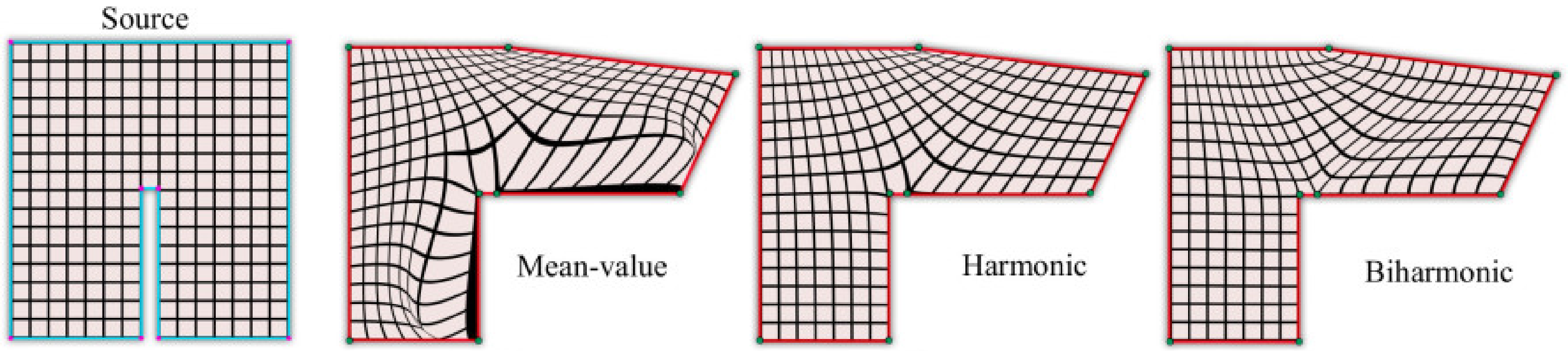
Cage creation

Building good cage is non-trivial task

Multiple works to avoid manual creation

Skeleton Based Cage Generation Guided by Harmonic Fields. S. Casti et al. Computer & Graphics 2019

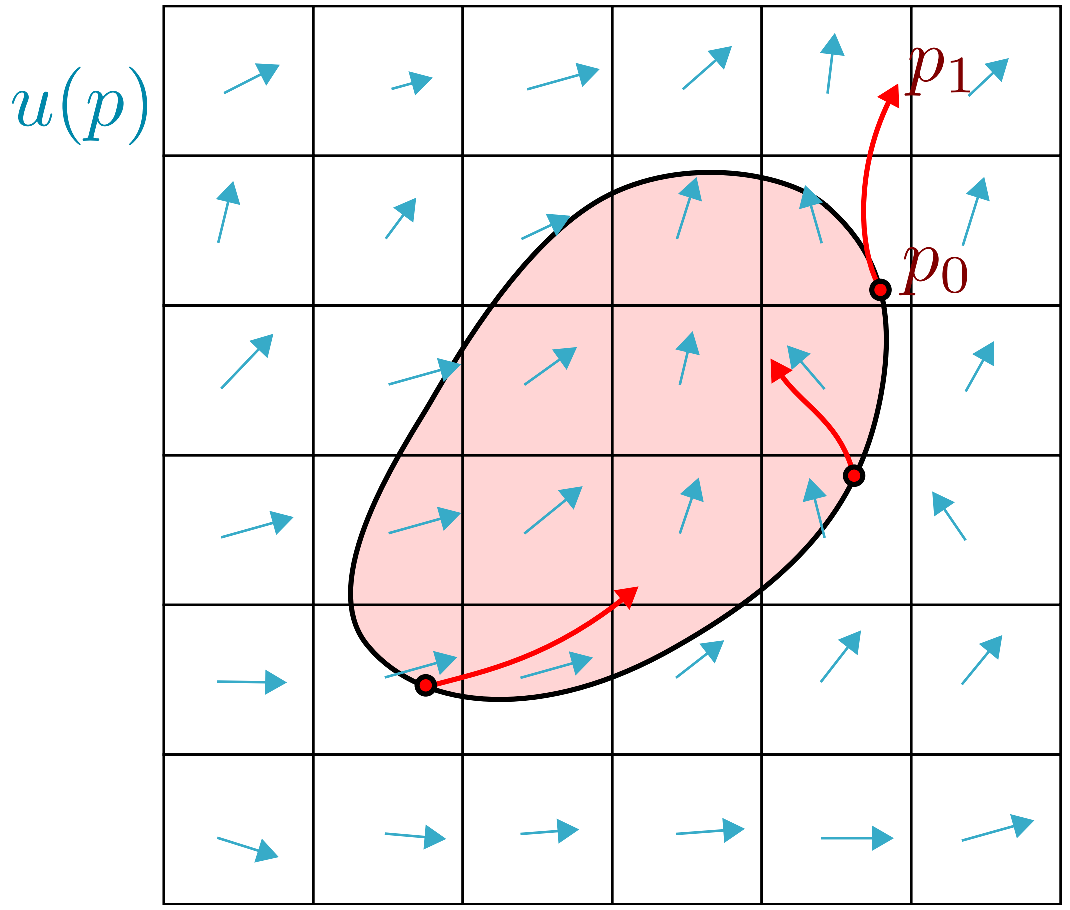
CageR: Cage-based Reverse Engineering of Animated 3D Shapes. J.-M. Thiery et al. CGF 2012.



Vector field-based deformation

Smooth vector field (or velocity) $u : (x, y, z) \to (u_x(x, y, z), u_y(x, y, z), u_z(x, y, z))$

At arbitrary time t, $\dot{p}(t) = u(p(t))$



Applying deformation = integrate vertex position along streamlines

$$p_1 = p_0 + \int_t u(p(t))\,\mathrm{d}t$$

At a given time t, the instantaneous deformation at position p is

$$f(p, t) = p + u(p, t) \to f = I + u$$

(+) If $\|u\| \neq 0$ (and $\neq$ singularity) no-intersection

Streamlines do not intersect

(-) Arbitrary vector fields hard to define and control.

Note: All physically-based deformation can be seen as vector field deformations

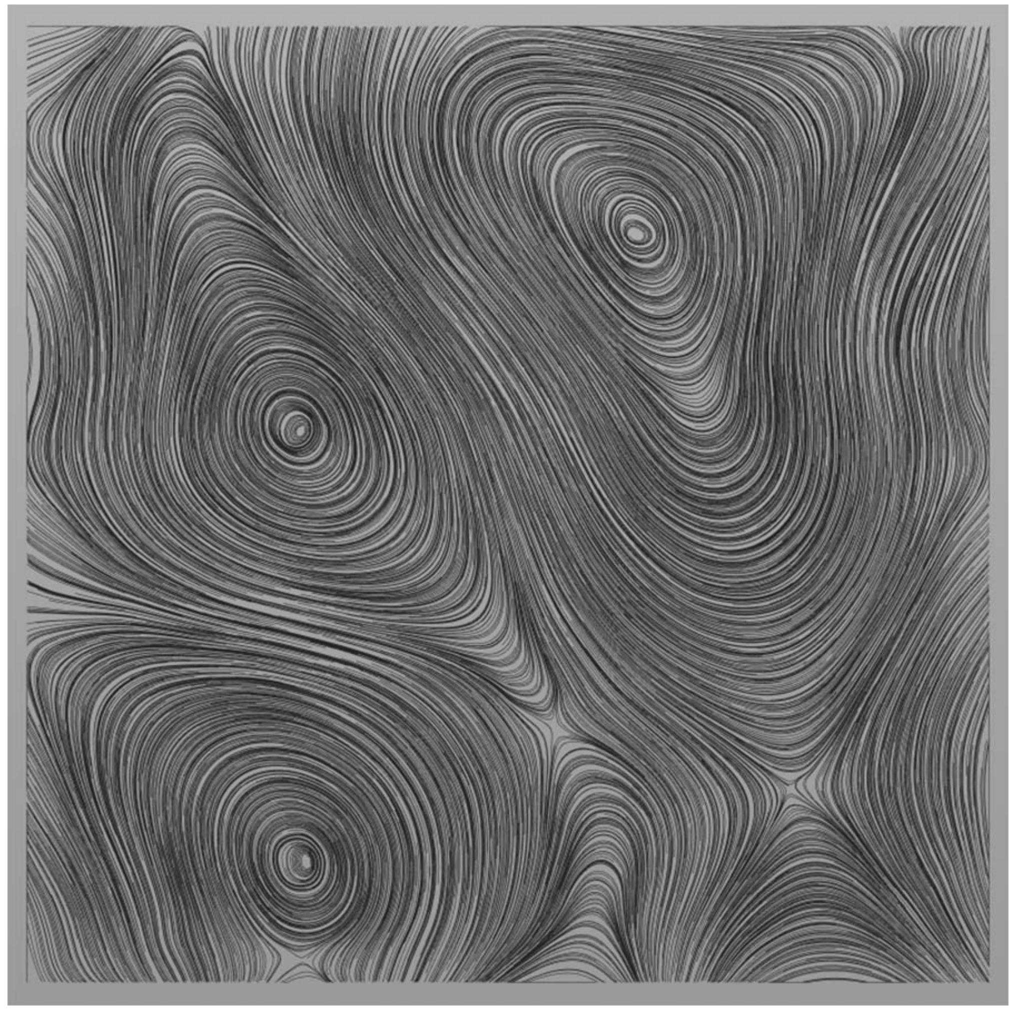
Divergence Free Vector Field

Divergence of u: $div(u) = \nabla \cdot u = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial u_y} + \frac{\partial u_z}{\partial u_z}$

Divergence free / solenoidal field : $div(u) = 0$

Ex. Magnetic fields, incompressible fluids velocity, are divergent free

$\Rightarrow$ Volume preserving deformation



Dem. (in 2D)

$$f = I + u$$

$$\Rightarrow J_f = \begin{pmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{pmatrix} = \begin{pmatrix} 1 + \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} \\ \frac{\partial u_y}{\partial x} & 1 + \frac{\partial u_y}{\partial y} \end{pmatrix}$$

Volume preserving deformation $det(J_f) = 1$

$$\Rightarrow \left(1 + \frac{\partial u_x}{\partial x}\right)\left(1 + \frac{\partial u_y}{\partial y}\right) - \frac{\partial u_x}{\partial y}\frac{\partial u_y}{\partial x} = 1$$

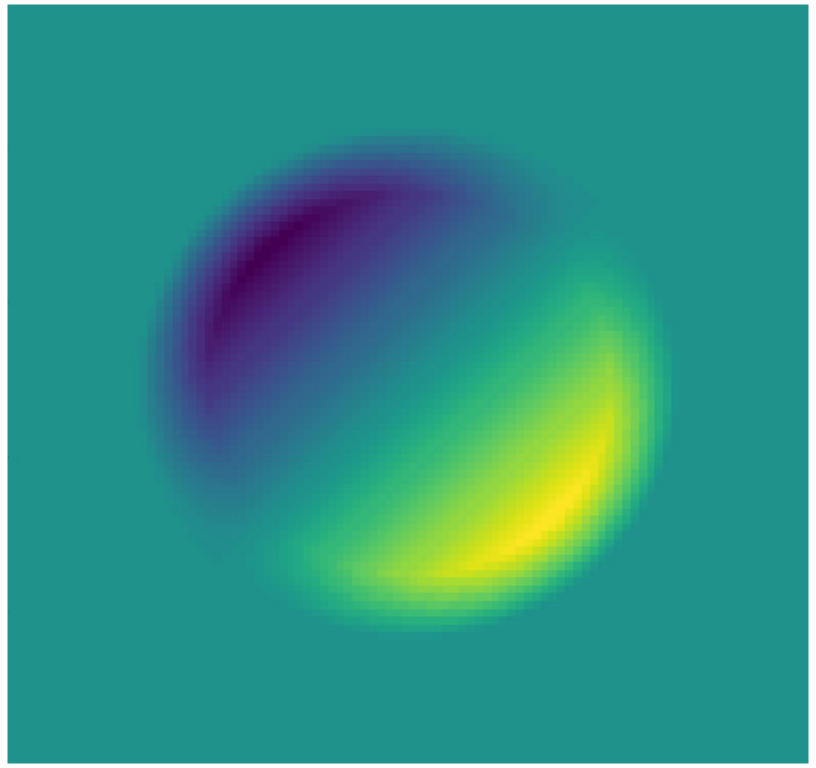
$$\Rightarrow \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0$$

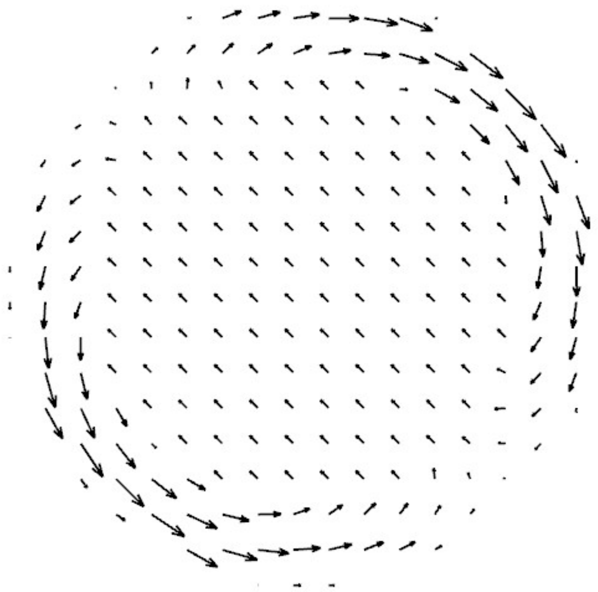
Building divergence free vector field

Example: Define 2 scalar fields (a, b)

Build vector field $u = \nabla a \times \nabla b$

$$div(u) = div(\nabla a \times \nabla b)$$
$$div(u) = \underbrace{curl(\nabla a)}_{=0} \cdot \nabla b - \nabla a \cdot \underbrace{curl(\nabla b)}_{=0}$$
$$div(u) = 0 \Rightarrow \text{divergence free vector field}$$



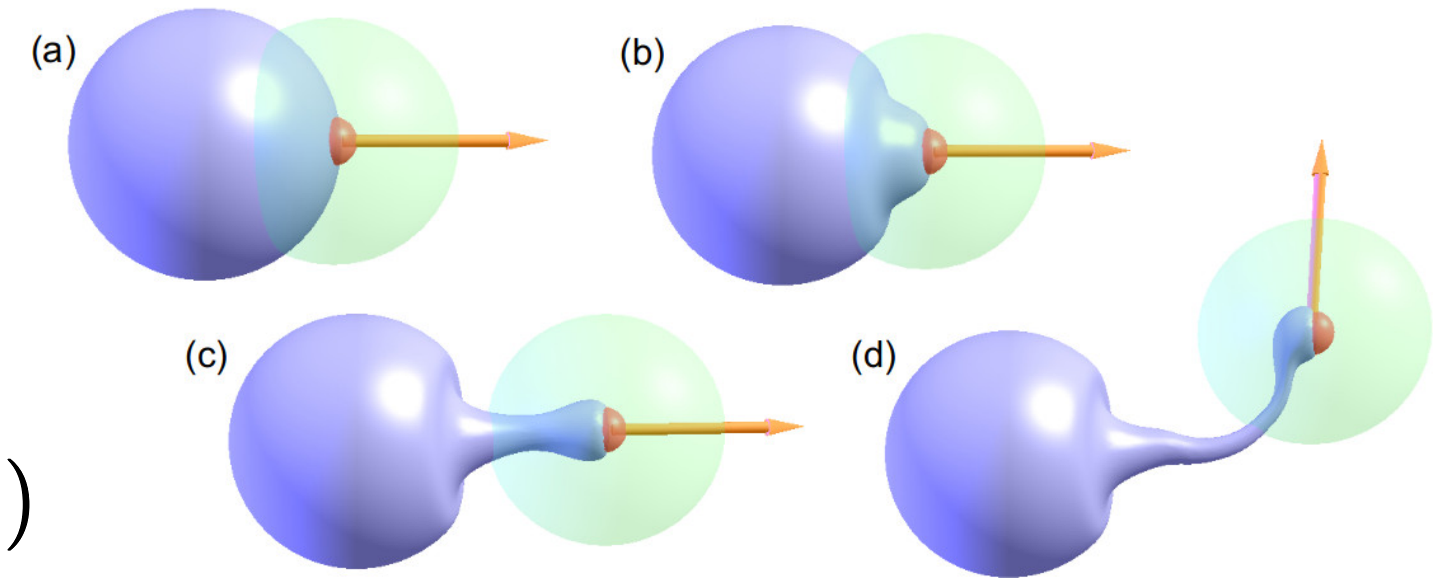


Divergence free vector field example

Example from [Vector Field Based Shape Deformations. W. von Funck et al. SIGGRAPH 2006]

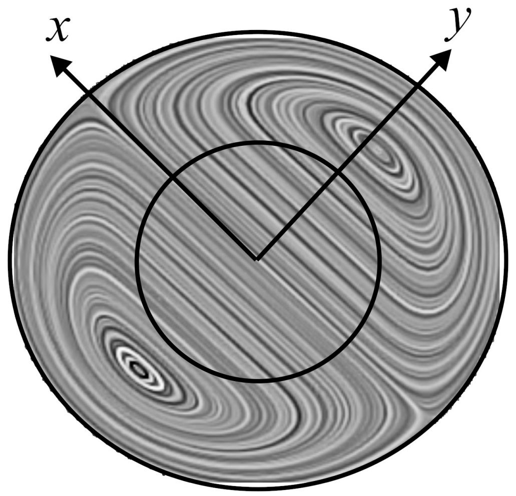
$$a(p) = \begin{cases} a_0(p) & \|p\| < r_1 \\ (1-\alpha(p))\, a_0(p) & r_1 \leq \|p\| \leq r_2 \\ 0 & otherwise \end{cases}$$

Similarily with $b(p)$



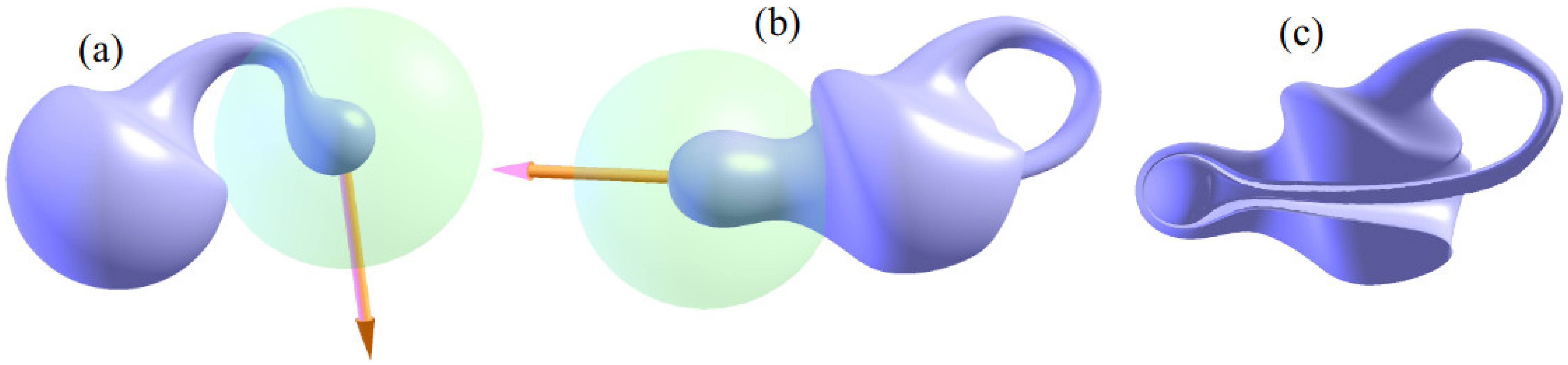
Translation: $a_0(p) = v_1 \cdot (p - p_0), b_0(p) = v_2 \cdot (p - p_0)$

v_1, v_2 arbitrary orthogonal unit vector to the direction of translation.

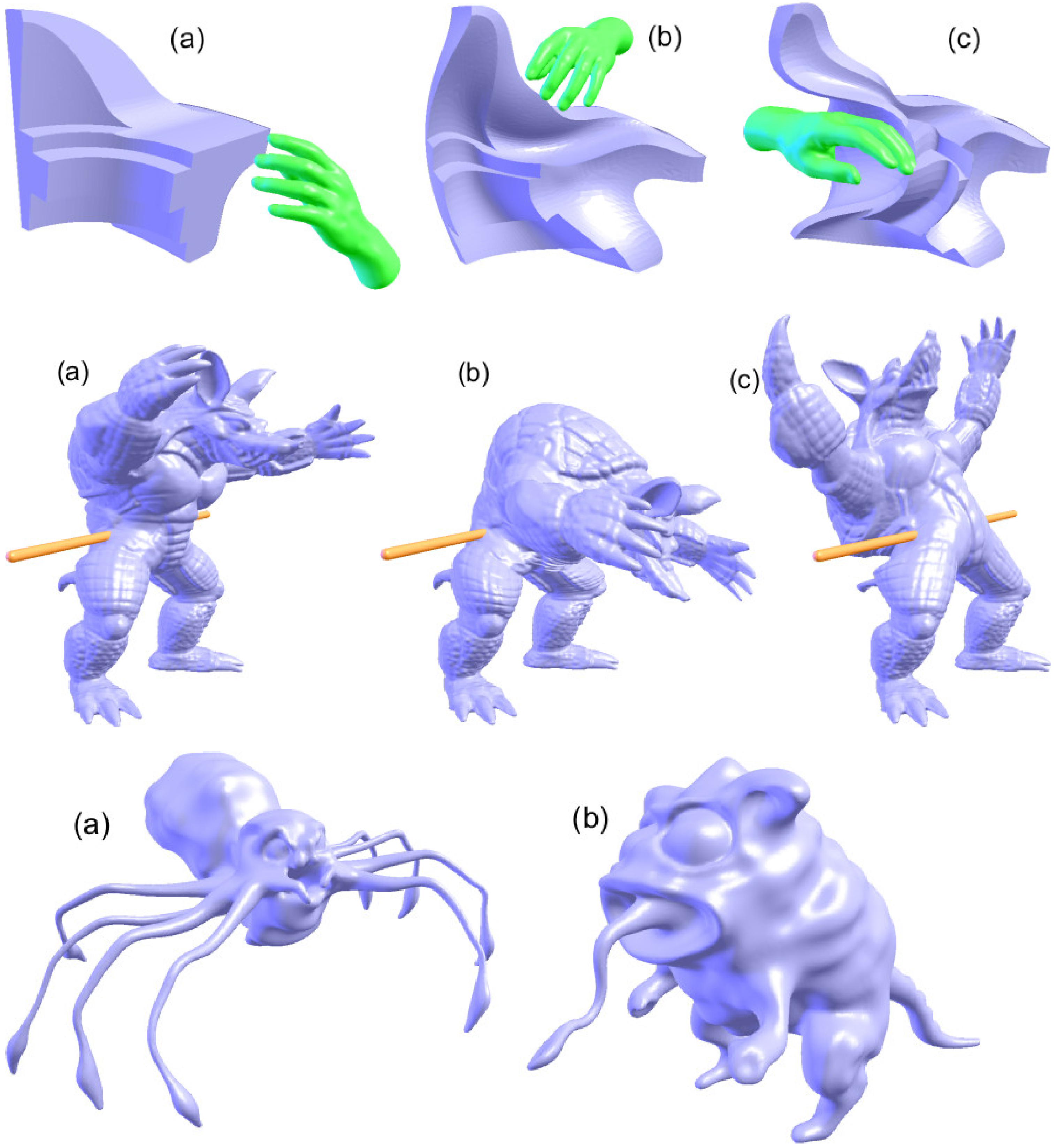


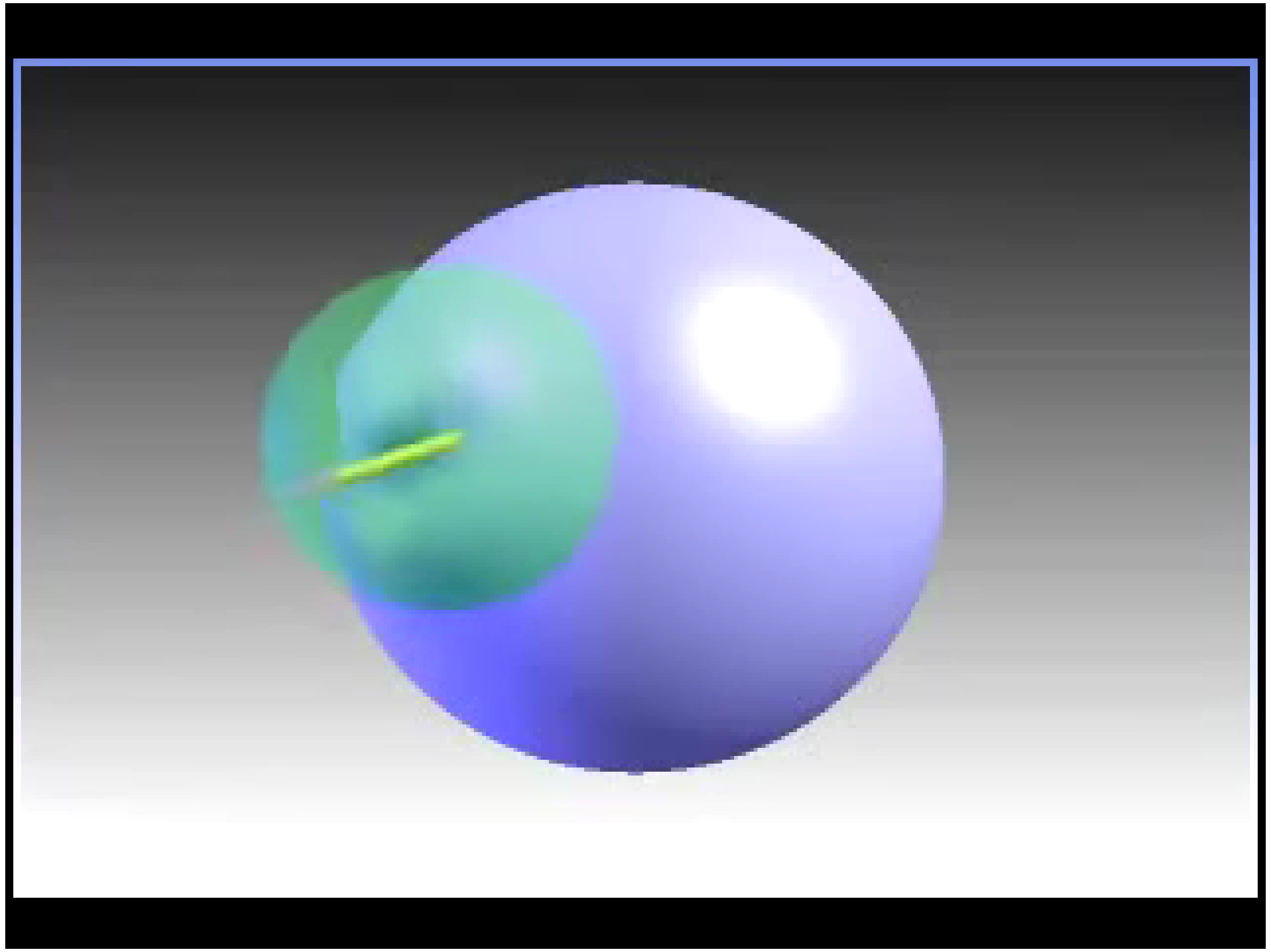
Bending: $a_0(p) = a \cdot (p - p_0), b_0 = \|a \times (p - p_0)\|^2$

a rotation axis.



Example of results





Case of implicit surfaces

Undeformed implicit surface $S = \{p \in \mathbb{R}^3 \mid \; g(p) = 0\}$

Given a vector field u

The implicit surface S' advected by u is defined as

$$\frac{\partial g}{\partial t} + u \cdot \nabla g = 0$$

Note: Only velocity normal to the surface changes the shape

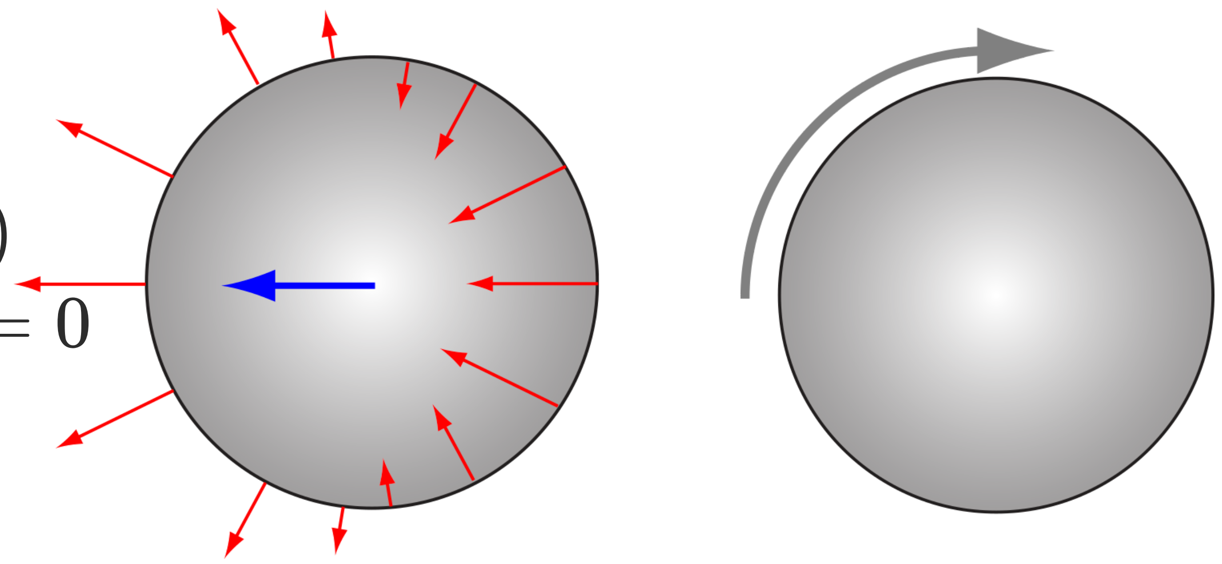
g is a space-time function $g(p,t)$

$g(p + u\mathrm{d}t, t + \mathrm{d}t) = g(p,t) = 0$

$\Rightarrow$ The differential $\mathrm{d}g = 0$

$\Rightarrow \frac{\partial g}{\partial t}\,\mathrm{d}t + \nabla g \cdot \mathrm{d}p = 0$

$\Rightarrow \frac{\partial g}{\partial t} + \nabla g \cdot \underbrace{\frac{\mathrm{d}p}{\mathrm{d}t}}_{u} = 0$



On the Velocity of an Implicit Surface