

Elastic models

Spring structure

Numerical solution of ODE

Cloth simulation

Material model

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Elasticity: Shape goes back toward its original rest position when external forces are removed.

- Purely elastic models don't lose energy when deformed (potential $\leftrightarrow$ kinetic)



Plasticity: Opposite of elasticity. Plastic material doesn't come back to their original shape (/change their rest position during deformation).

- Ductile material - can allow large amount of plastic deformation without breaking (plastic)
- Brittle - Opposite (glass, ceramics)

Viscosity: Resistance to flow (usually for fluid, ex. honey)

In reality

- *Elasto-plastic materials:* Allow elastic behavior for small deformation, and plastic at larger one.
- *Visco-elastic materials:* Elastic properties with delay.

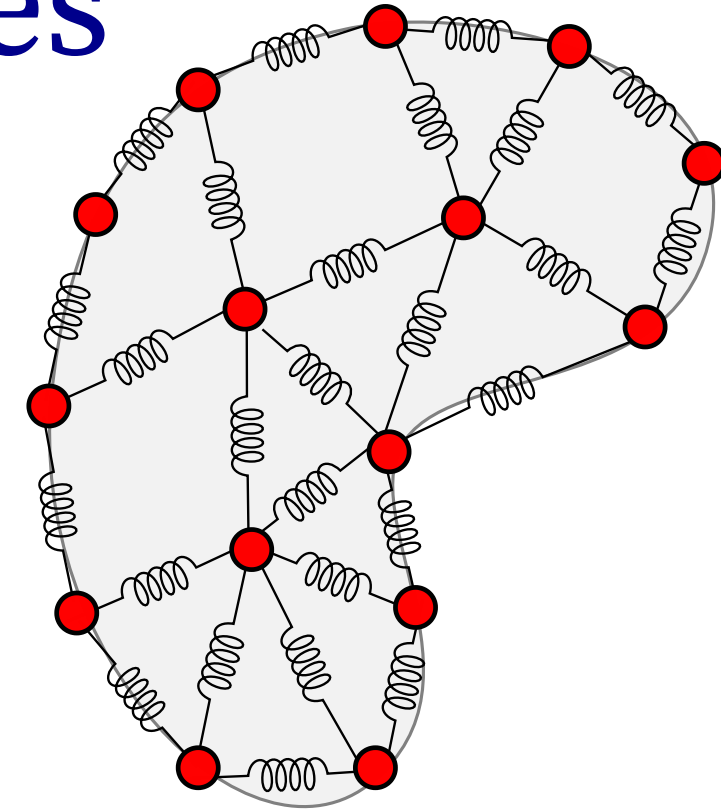


Modeling elastic shapes with particles

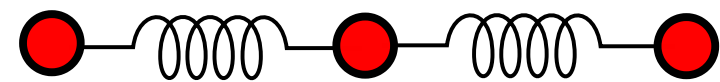
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Spring mass systems

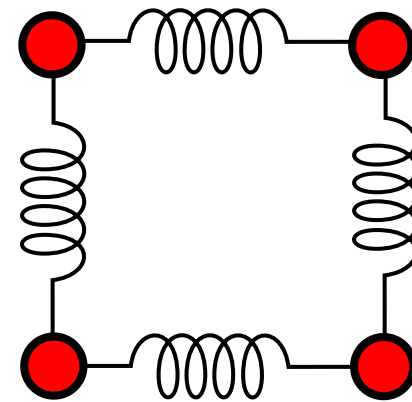
- Particles (position, velocity, mass): samples on shape
- Springs : link closed-by particles in the reference shape



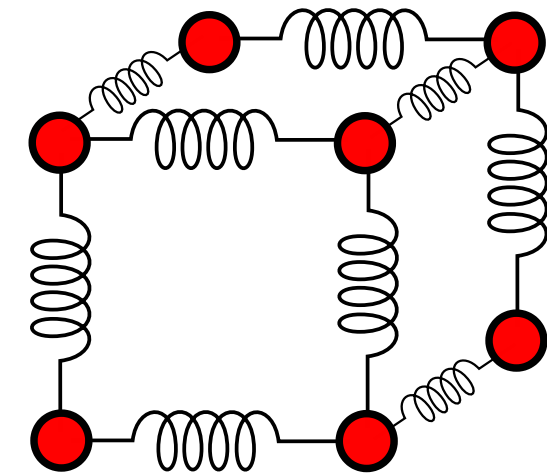
1D curve structure



2D surface structure



3D volume structure



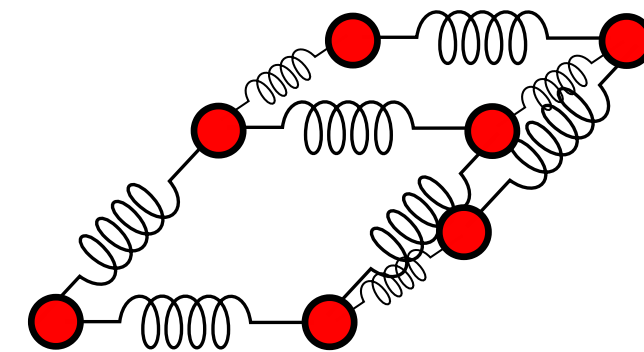
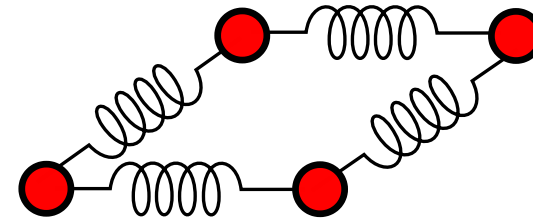
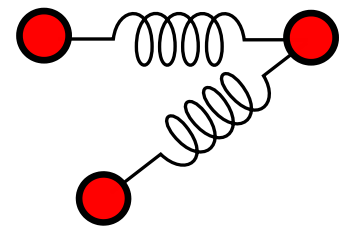
Spring structure

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How to model spring connectivity ?

- **Structural springs:** 1-ring neighbors springs ($\simeq$ mesh edges)

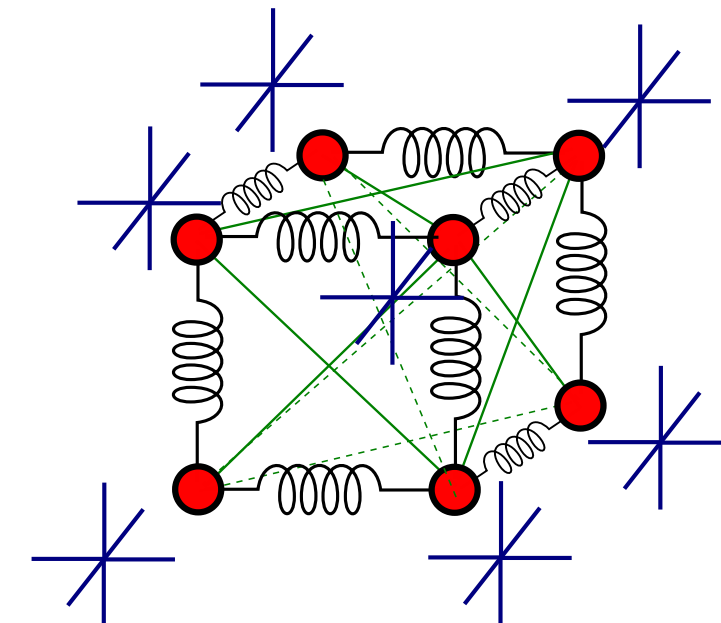
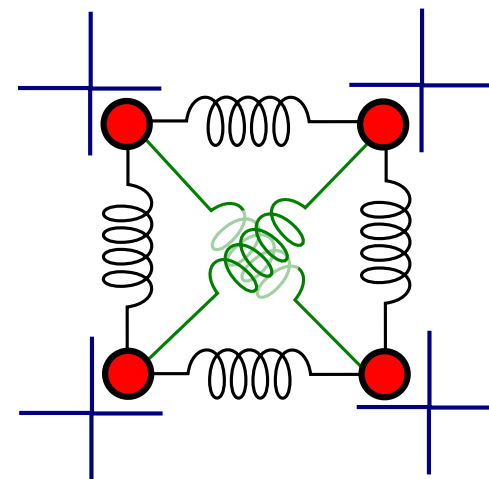
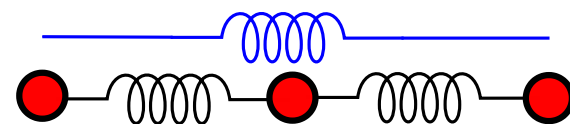
(+) Limit elongation/contraction, (-) Allows shearing, and bending



$\Rightarrow$ Add extra springs connectivity

- **Shearing springs:** Diagonal links

- **Bending springs:** 2-ring neighborhood

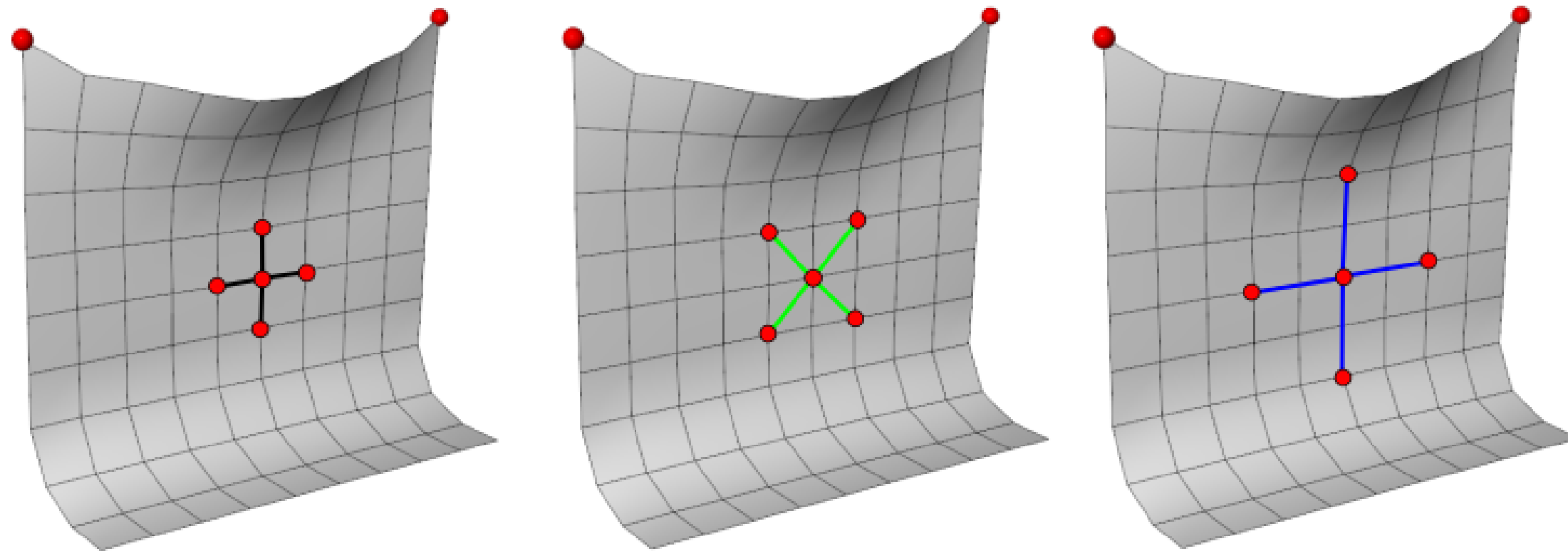


Cloth Simulation

Mass-spring cloth simulation

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- Particles are sampled on a $N \times N$ grid.
 - Each particle has a mass m ($m_{cloth} = N^2 m$)
- Set structural, shearing and bending springs.



Forces

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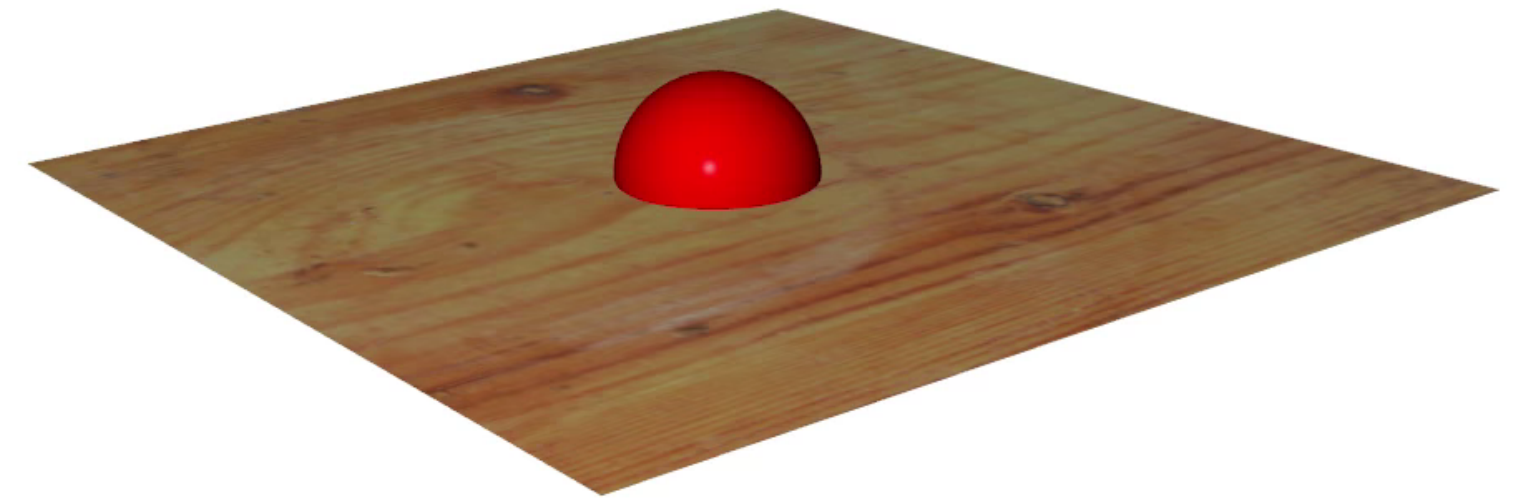
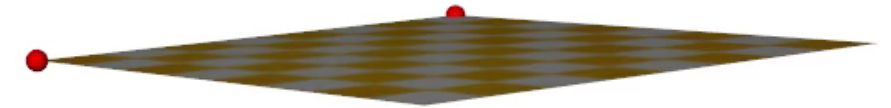
- On each particle: gravity + drag + spring forces

$$- F_i(p, v, t) = m_i g - \mu v_i(t) + \sum_{j \in \mathcal{V}_i} K_{ij} \left(\|p_j(t) - p_i(t)\| - L_{ij}^0 \right) \frac{p_j(t) - p_i(t)}{\|p_j(t) - p_i(t)\|}$$

- $\mathcal{V}_i$: neighborhood of particle i

- L_{ij}^0 : rest length of spring ij

Associated ODE $\forall i, \begin{cases} p_i'(t) = v_i(t) \\ v_i'(t) = F_i(p, v, t)/m_i \end{cases}$

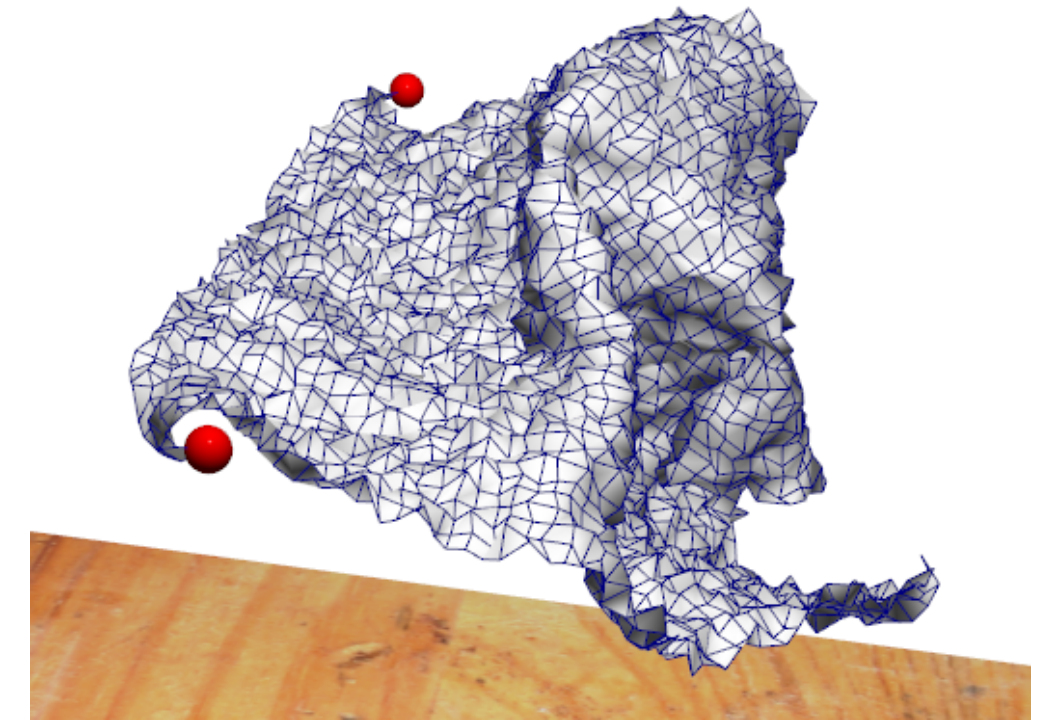


Q. How can we model the effect of the wind ?

Note on Mass-Spring numerical solution

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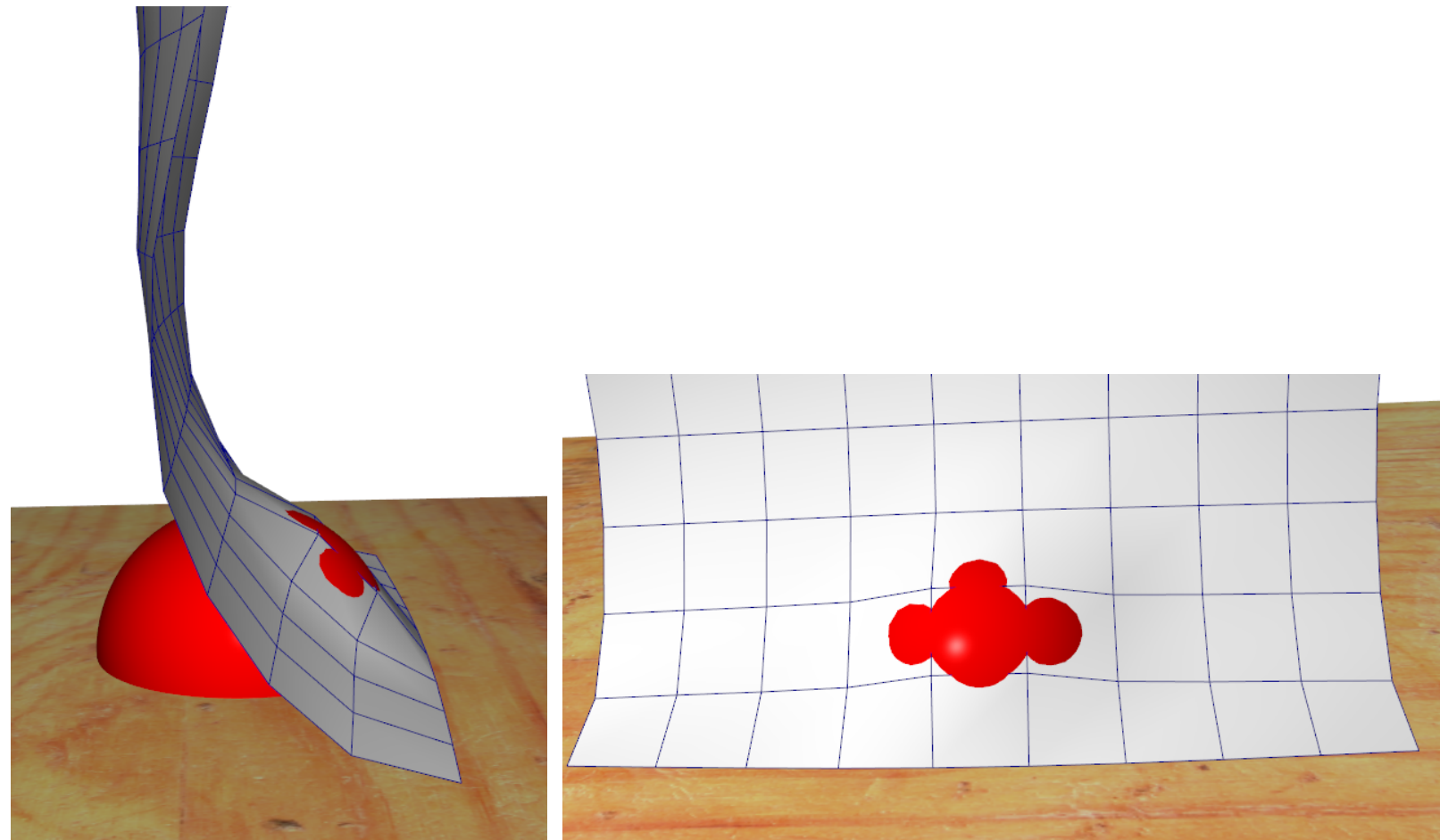
- Non-linear ODE
- Large K_{ij} : good length preservation, but stiff ODE
⇒ divergence of explicit schemes.
- Avoid explicit Euler (divergence)
- Semi-implicit Euler/Verlet works fine for low K_{ij}
Semi-implicit Euler + PBD allows simple integration + stable stiff springs
[Muller et al. [PBD](#) , [Inextensible clothing in Computer Games](#)]
- RK4 more accurate (but higher complexity than Verlet)
- Implicit Euler : requires linearization, but very stable



Collisions

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- Simple approach : Handled as collision between particles and shapes
 - (+) Simple and efficient
 - (-) Collision may still appears within a triangle
- ⇒ Exhaustive approach: edges + faces



Limitation of mass spring model and continuous model

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- Does mass-spring system converge toward a unique solution when sampling increase ?

⇒ No :(

Depends on the connectivity → bad for physical accuracy

Corollary

- Mass-springs work well for grid-mesh structure (draping)
- Less for arbitrary triangular meshes

1st improvment: Change toward energy formulation for bending springs (limits locking effect)

$$F = \frac{\partial E}{\partial p}$$

$$E = \frac{1}{2} K L \kappa^2, \kappa: \text{curvature}$$

[Cho et al, Stable but Responsive Cloth, ACM SIGGRAPH 2002]



Triangle as continuous elements

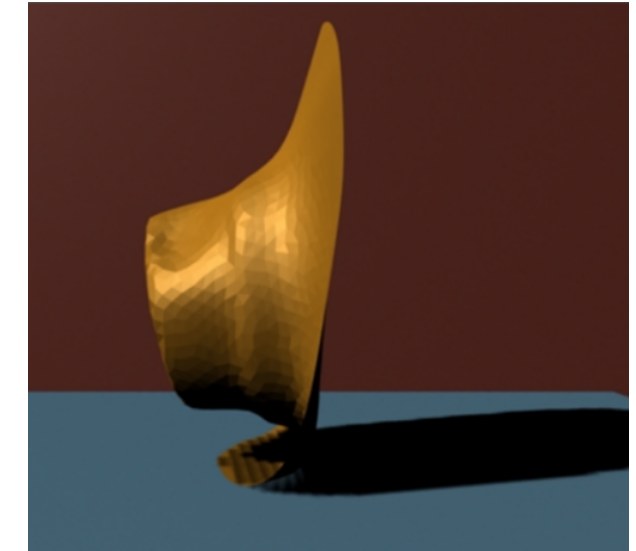
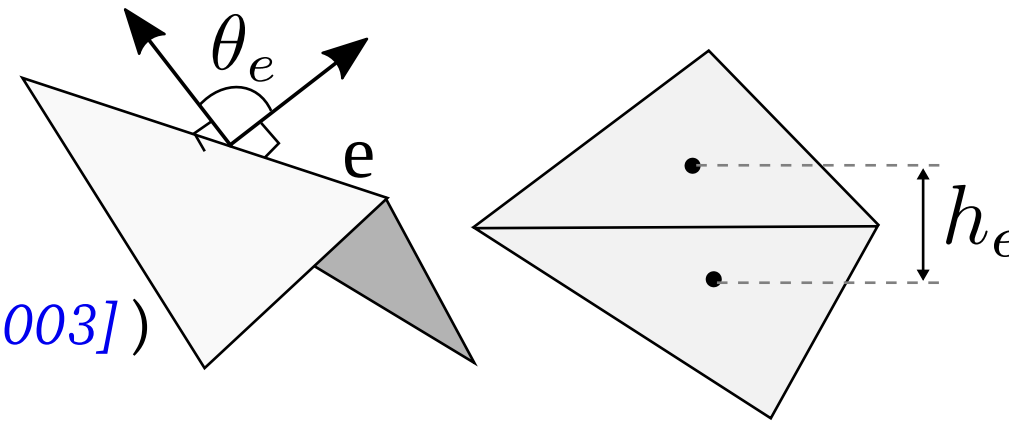
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- Defining Bending Energy between triangles

$$- W_B(x) = \sum_{\text{edges } e} (\theta_e - \theta_e^0) \frac{\|e^0\|}{h_e^0}$$

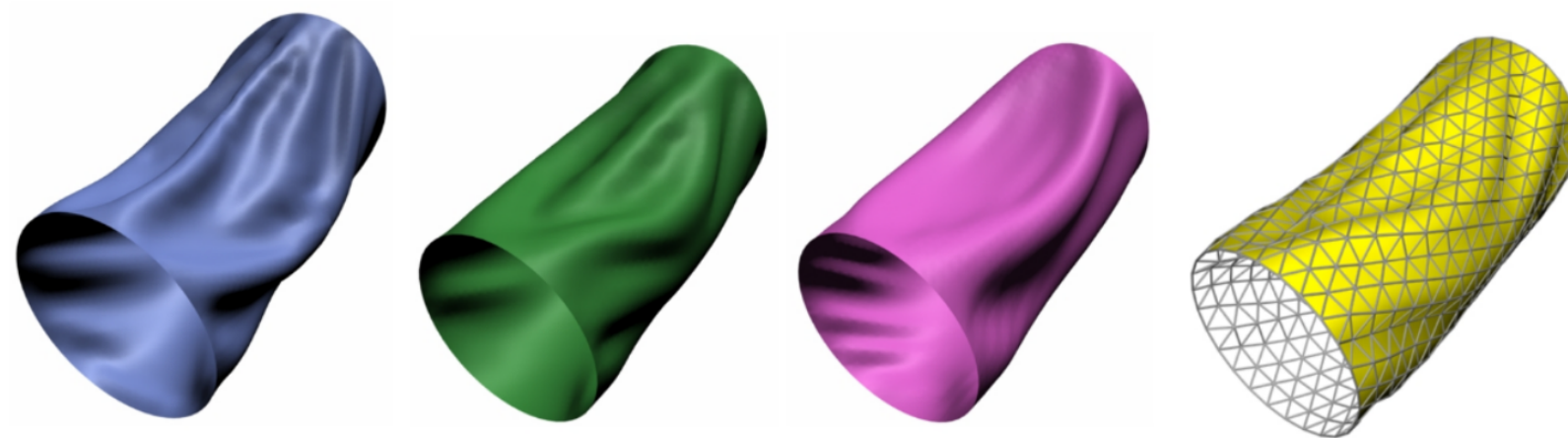
[E. Grinspun et al., Discrete Shells, SCA 2003]

(or expressed using forces in [R. Bridson et al., SCA 2003])



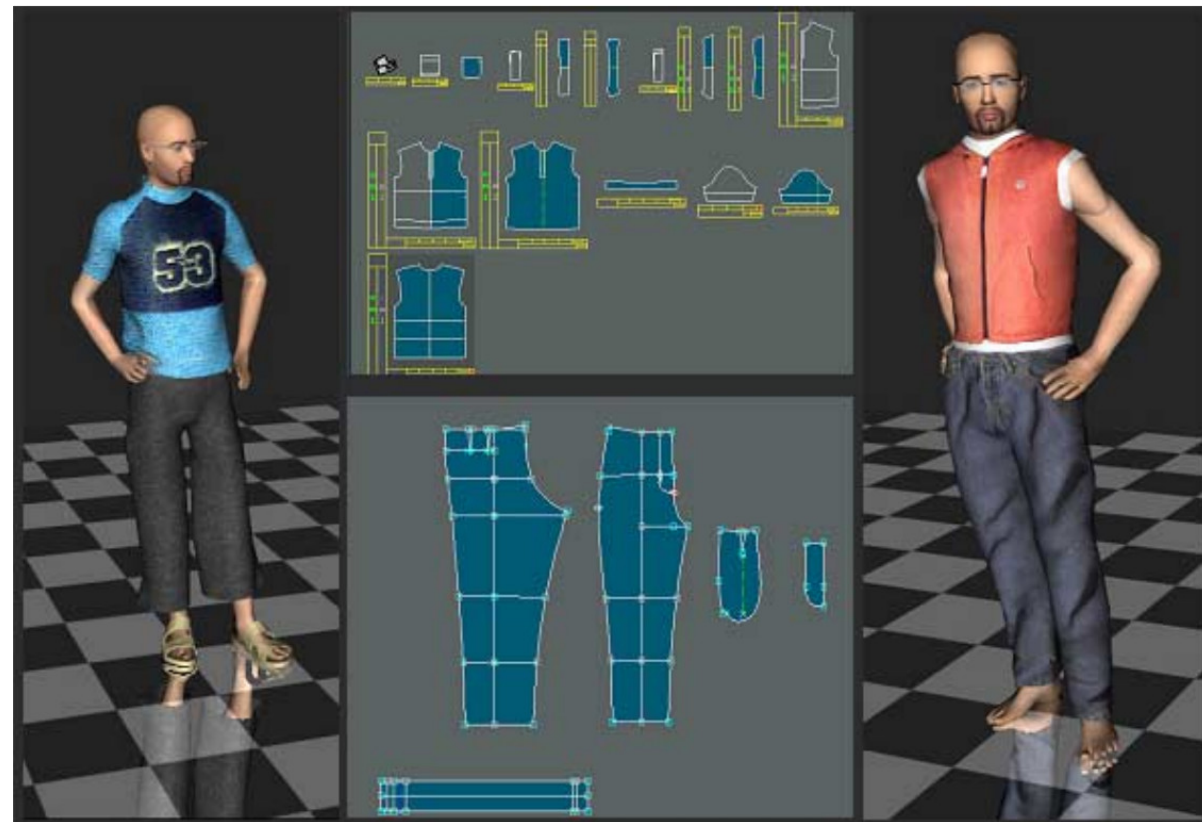
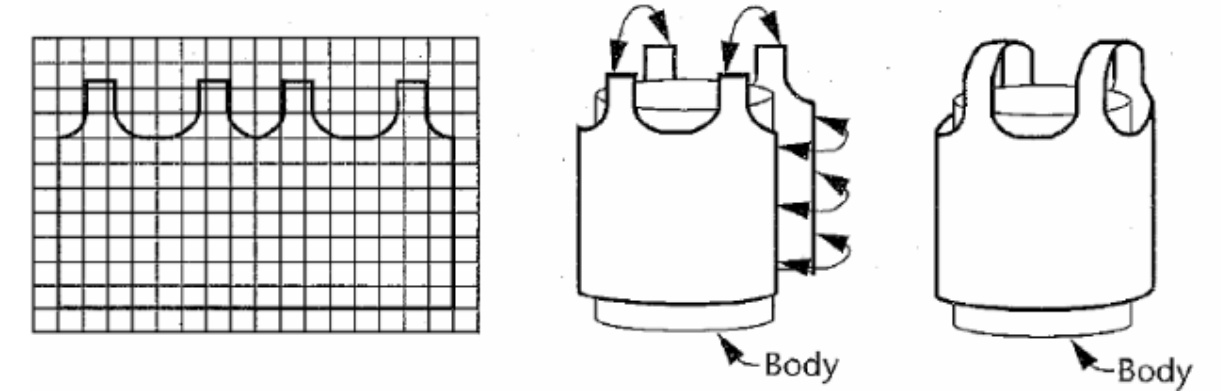
- Going toward full FEM numerical resolution

- B. Thomaszewski et al. [SCA 2006], [VRIPHYS 2008], [EG 2009].



Clothing

- Stitch 2D patterns together to generate full cloth
- Cloths are developable material (preserve length w/r their 2D patterns)

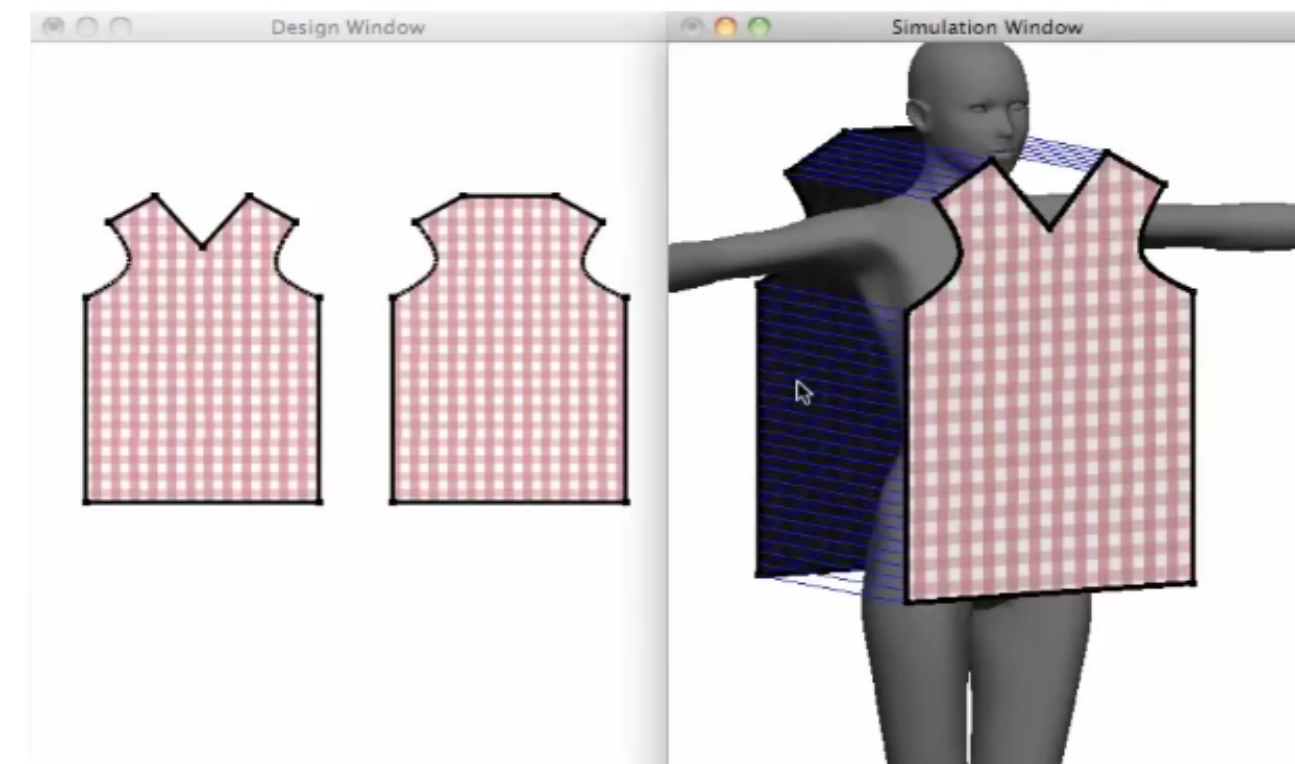


[Thalman et al. 2002]

Overview

Cloth Pattern

Cloth Simulation



[Umetani et al., 2011]

Detecting self collision

Handled as moving point in collision with moving triangle

Inputs

- Triangle $P_1(t)P_2(t)P_3(t)$, a point $P(t)$
- Each position $P_k(t) = P_k(0) + t v_{P_k}$

Computing intersection

Necessary condition

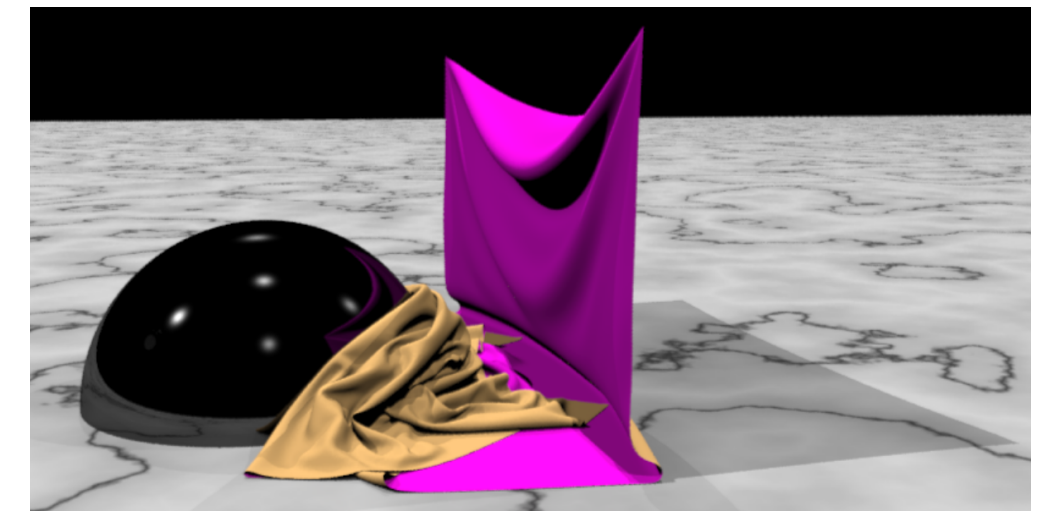
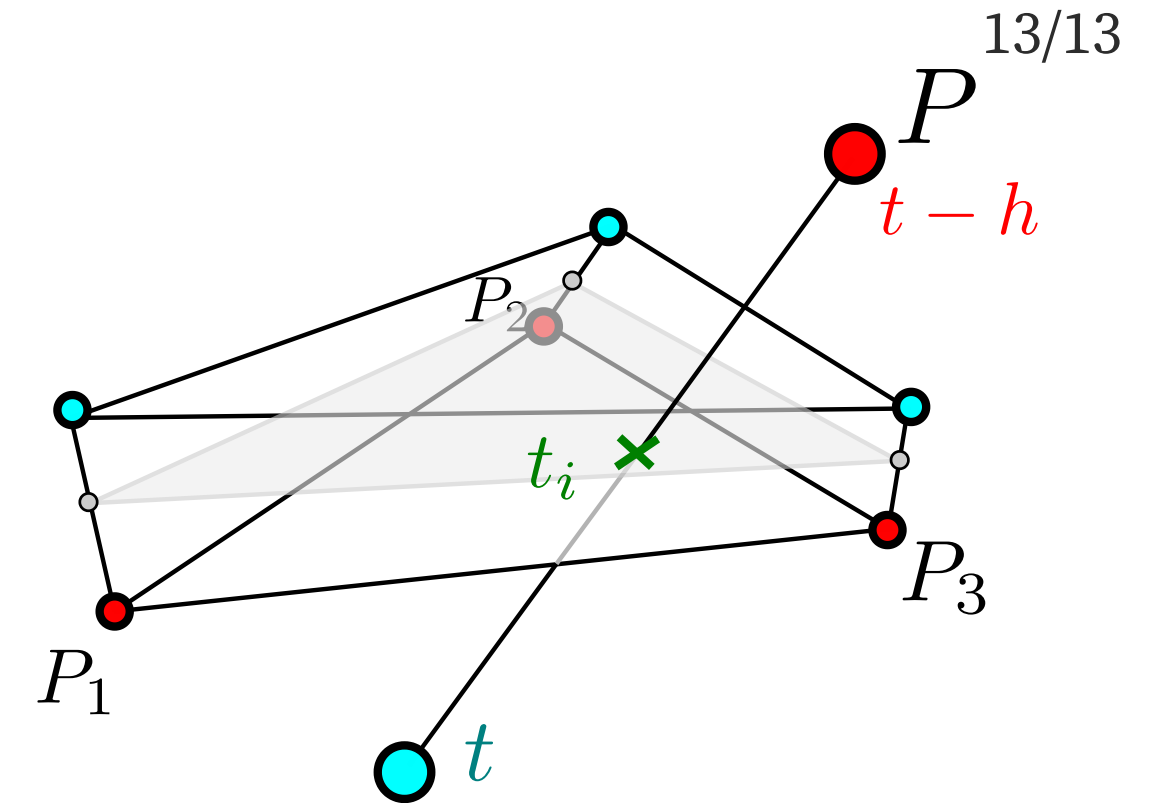
- Find $t_i \in [0, h]$ such that $P(t_i)$ is in triangle plane
 $(P(t_i) - P_1(t_i)) \times n(t_i) = 0$
 $n(t_i)$: normal of the triangle at time t_i

Sufficient condition

- Check $P(t_i)$ is inside the triangle

$$P(t_i) = \alpha P_1(t_i) + \beta P_2(t_i) + \gamma P_3(t_i)$$

$$(\alpha, \beta, \gamma) \in [0, 1]^3, \alpha + \beta + \gamma = 1$$



[X. Provot. Collision and self-collision handling in cloth model dedicated to design garments. Graphics Interface 1997.]

[R. Bridson et al. Robust Treatment of Collisions, Contact and Friction for Cloth Animation. ACM SIGGRAPH 2002]