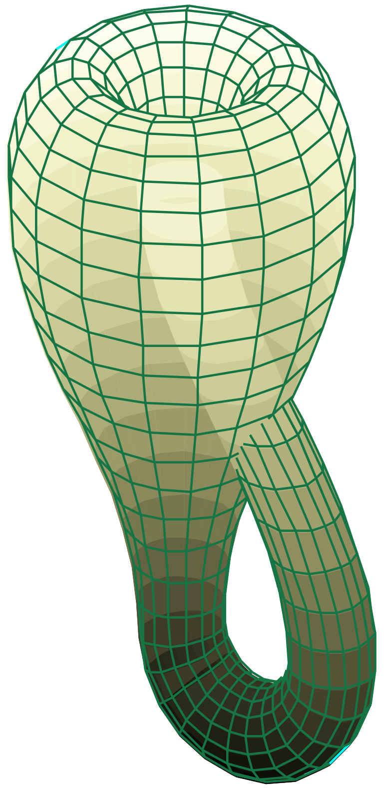
MPRI 2.39 - Modeling with Implicit Surfaces

Limits of boundary representations modeling

B-Rep



Meshes, Parametric surfaces, Subdivision surfaces, etc.

- Topology modification hard to integrate
- Boundary surface doesn't necessarily represents a volume
  ex. Klein bottles, self-intersections
  See *Combinatorial Maps* for robust encoding of Brep representation.

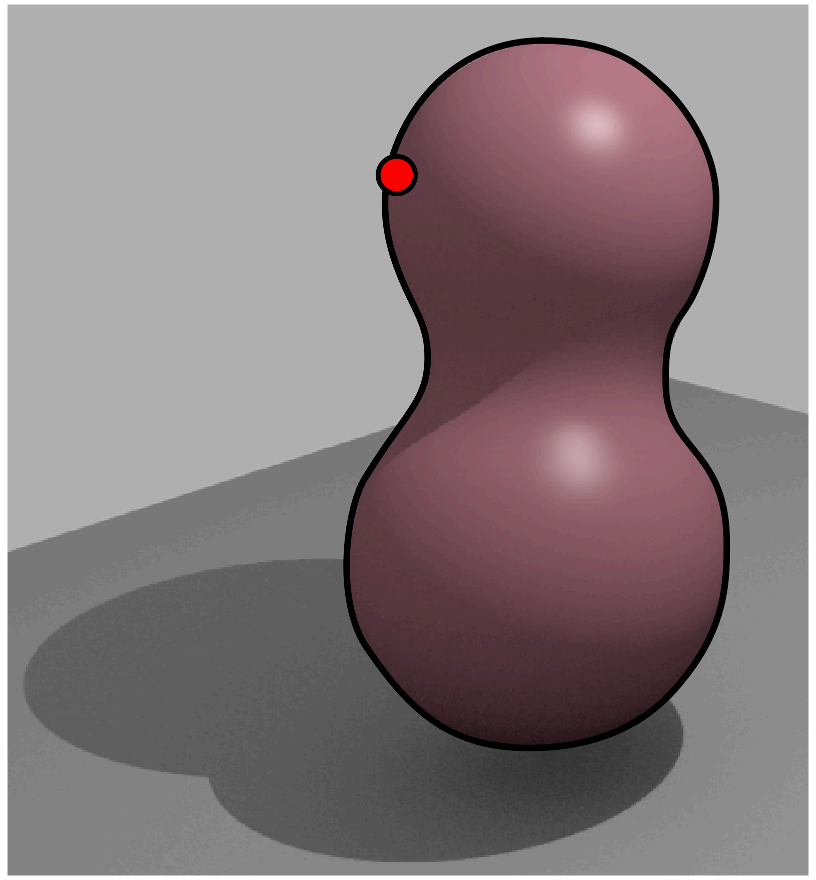
Implicit surfaces

$S = \{(x, y, z) \in \mathbb{R}^3 \mid f(x, y, z) = iso\}$

Intrinsically based on volumetric representation

- Robust volume modeling
- Easy blending/merging between pieces

Properties of implicit surfaces



$S = \{p \in \mathbb{R}^3 \mid f(p) = iso\}$

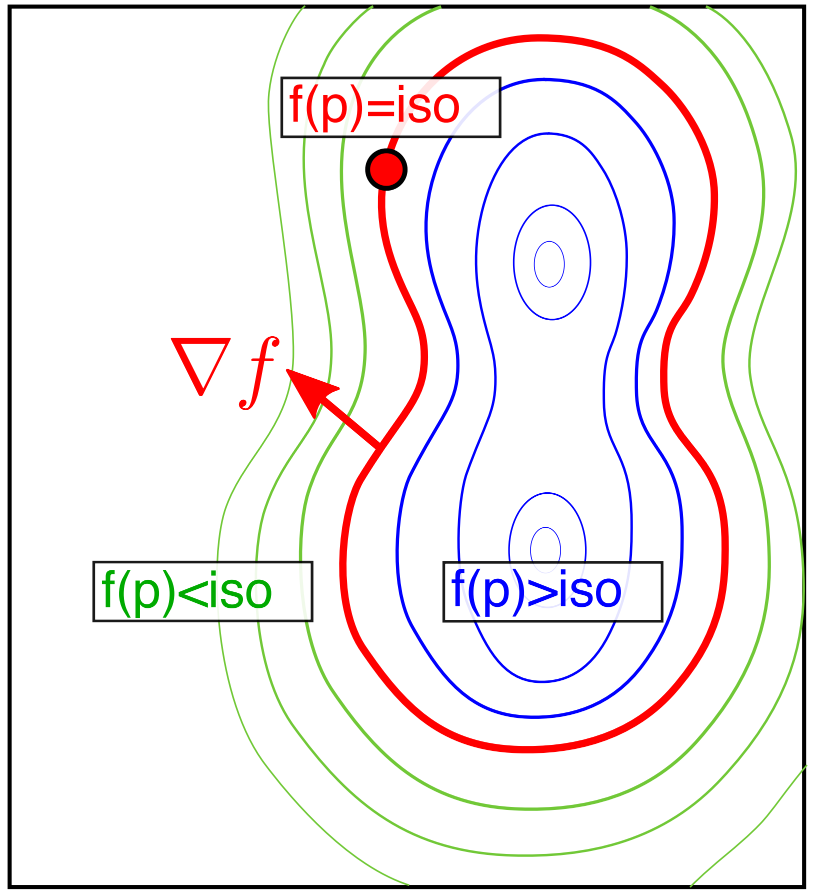
- Position p on the surface : $f(p) = iso$
- Position p
  - inside the surface $f(p) > iso$
  - outside the surface $f(p) < iso$ (convention)
  - => *Easy query for interior/exterior position*
- Normal on the surface $n = \nabla f(p)$

Consider dp infinitesimal vector within the tangent plane of the surface at point p

$f(p) = f(p + dp) = iso$

$\Rightarrow f(p) - f(p + dp) = 0$

$\Rightarrow dp \cdot \nabla f(p) = 0$



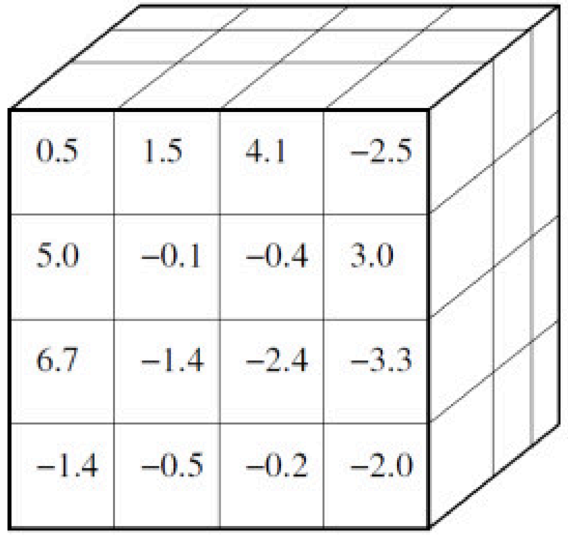
Encoding scalar field

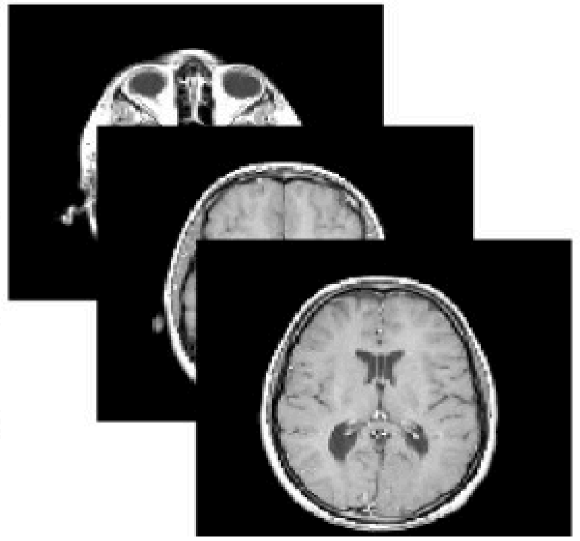
Discrete representation

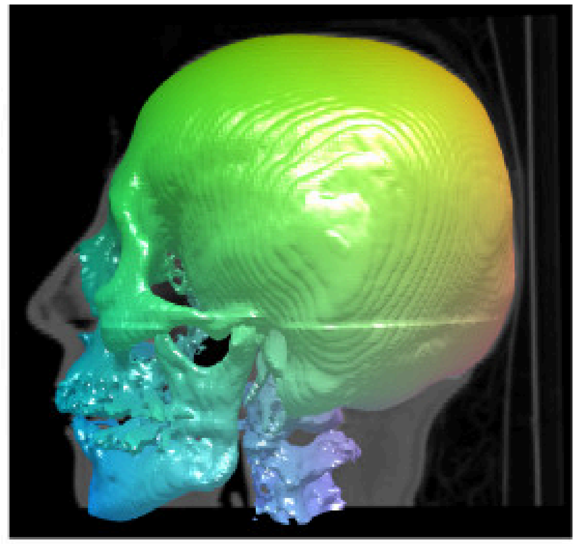
Grid based: one value per voxel

Naturally obtained from scanner acquisition

(High memory cost)



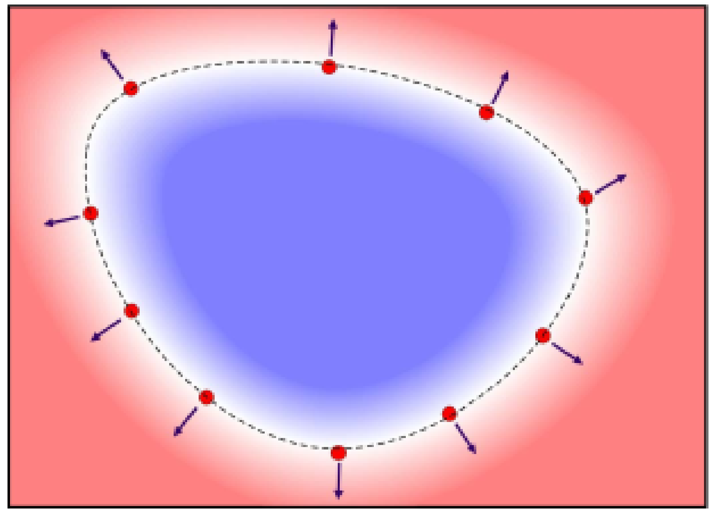




Continuous representation

From optimization (ex. Radial Basis Functions)

User defined (ex. Skeleton based)



Blobs

General idea (1982)

- Decreasing function f_i of distance to a point p_i
- p_i is called a skeleton

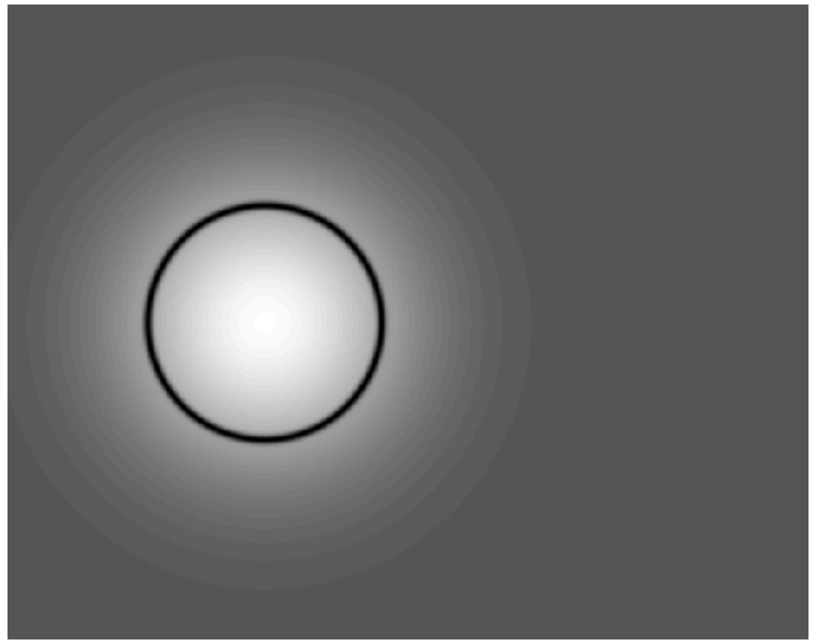
- Exemple **Exponential field**

$$f_i(p) = K_i \exp\left(-\frac{\|p - p_i\|^2}{R_i^2}\right)$$

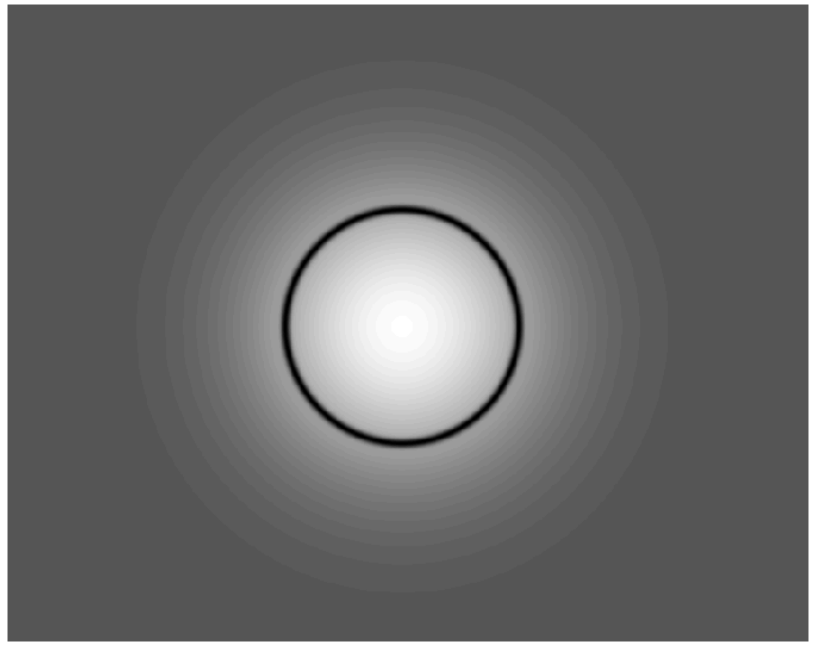
Blinn 1982

- Consider $K_0 = 1, R_0 = 1, p_0 = (0.0, 0.0, 0.0)$, what is $f_0(p) = 0.5$?
- Consider $K_1 = 1, R_1 = 1, p_1 = (1.0, 0.0, 0.0)$, what is
  $\max(f_0(p), f_1(p)) = 0.5$?
  $\min(f_0(p), f_1(p)) = 0.5$?
- Consider $K_1 = 1, R_1 = 1, p_1 = (2.0, 0.0, 0.0)$, what is
  $f_0(p) + f_2(p) = 0.5$?

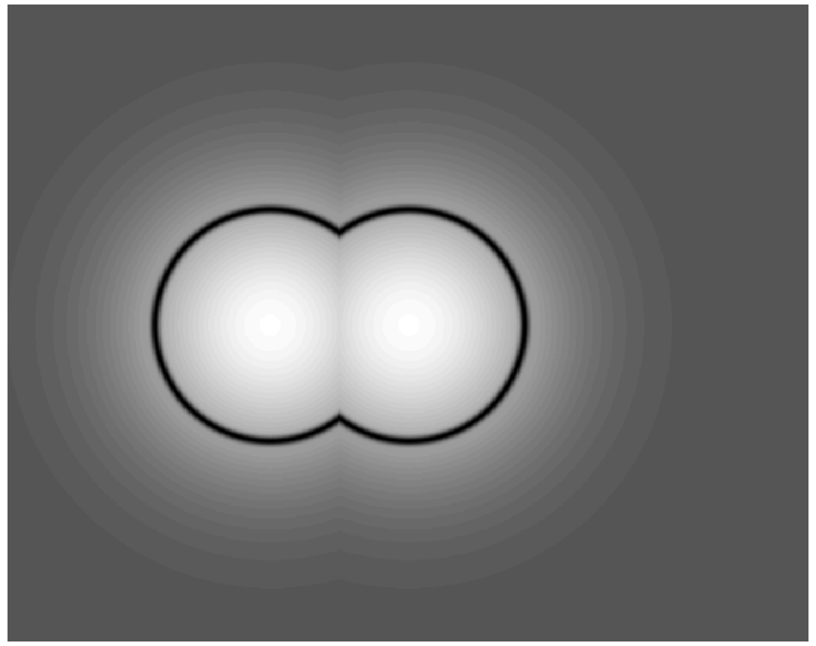
Blobs max / min



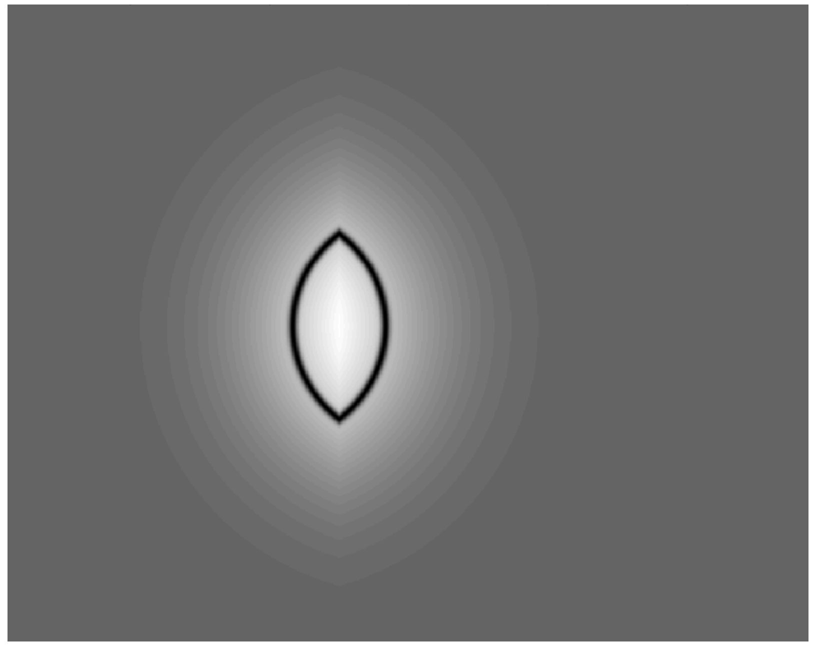
f_0



f_1

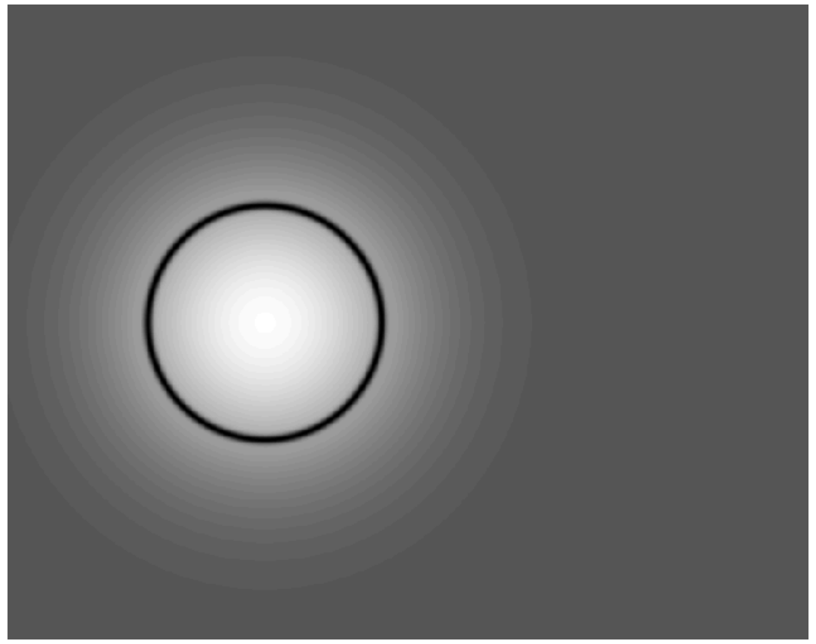


$\max(f_0, f_1)$

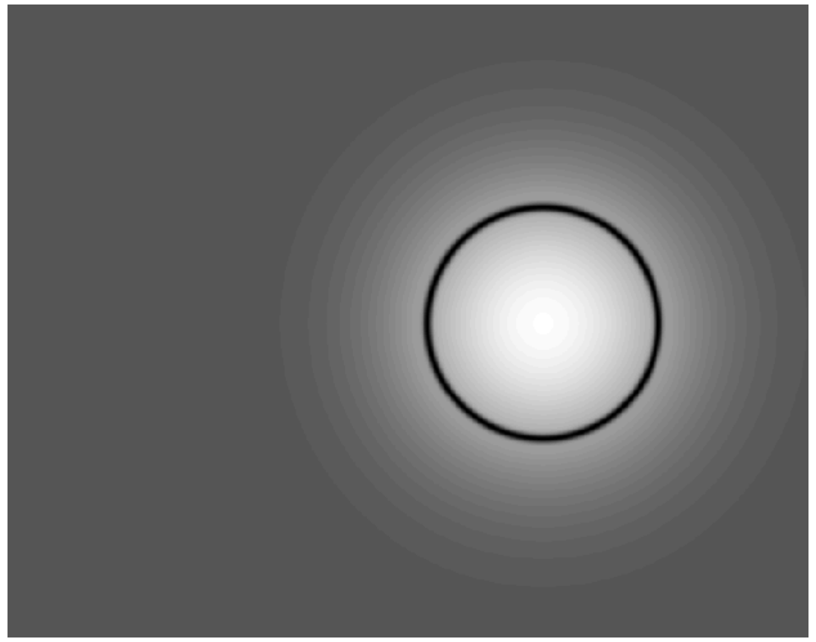


$\min(f_0, f_1)$

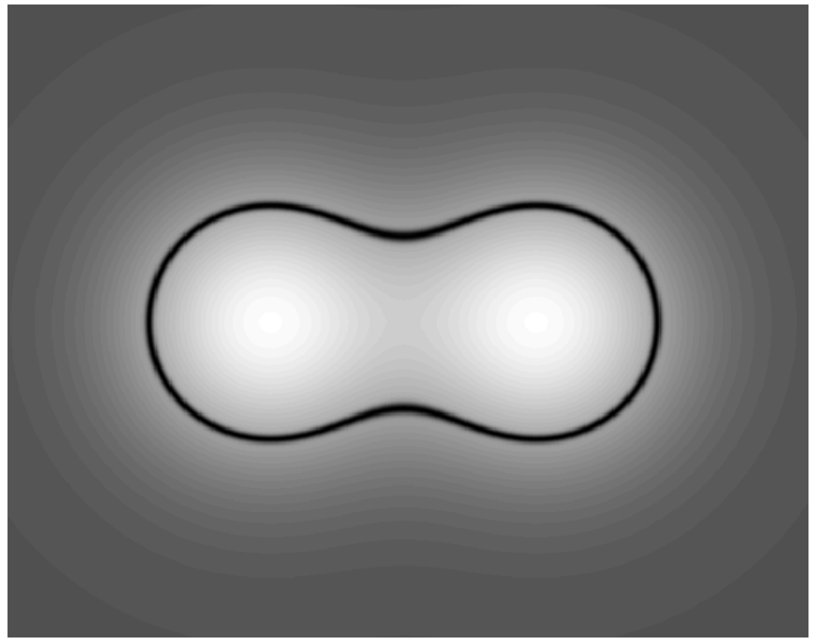
Blobs sum



f_0



f_2



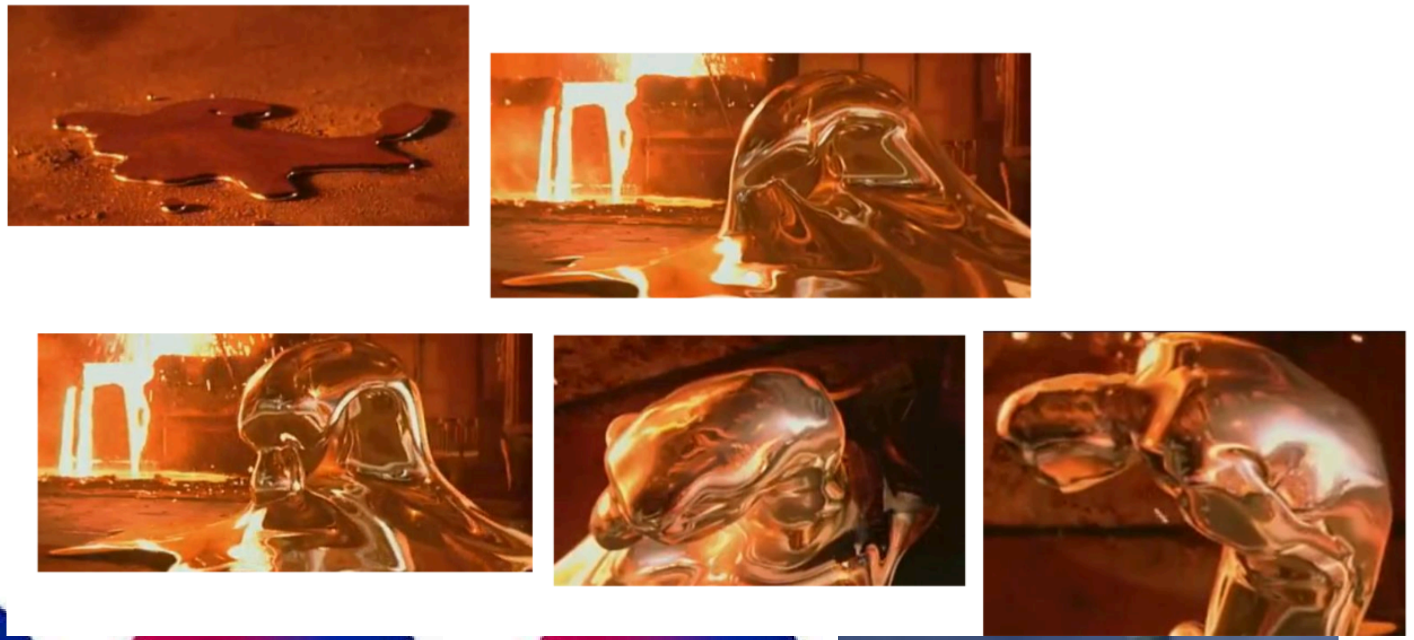
$f_0 + f_2$

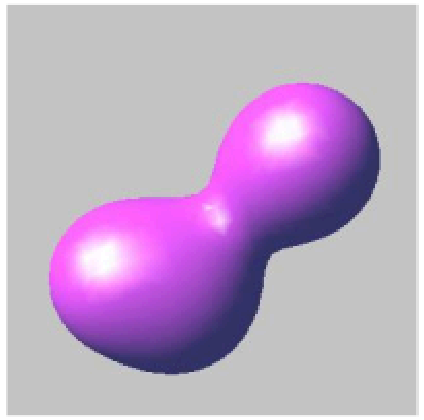
Function blending

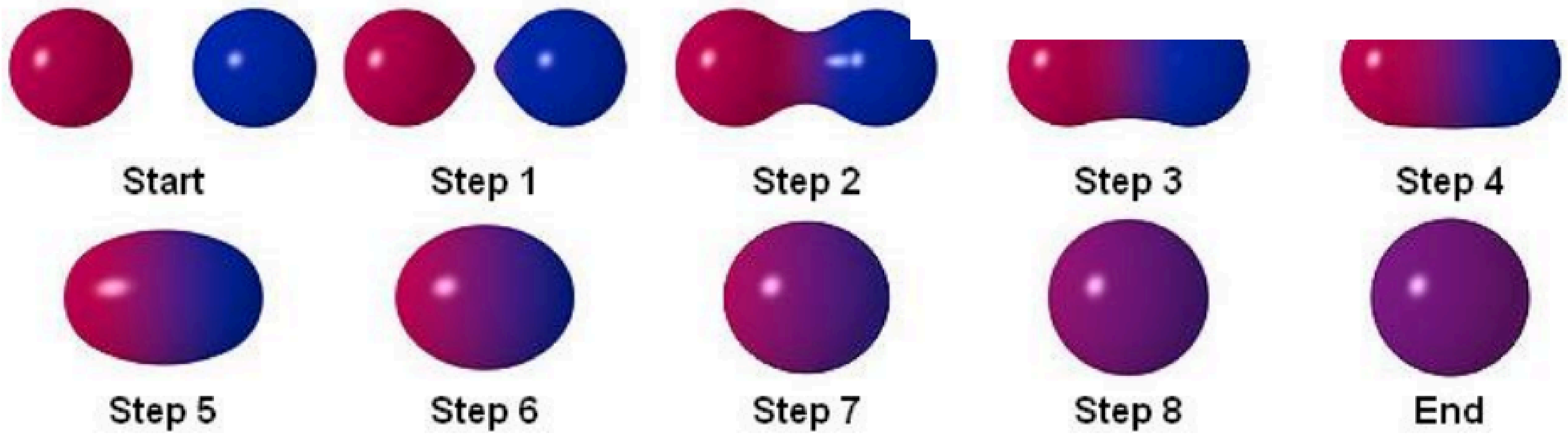
$$f_i(p) = K_i \exp\left(-\frac{\|p - p_i\|^2}{R_i^2}\right)$$

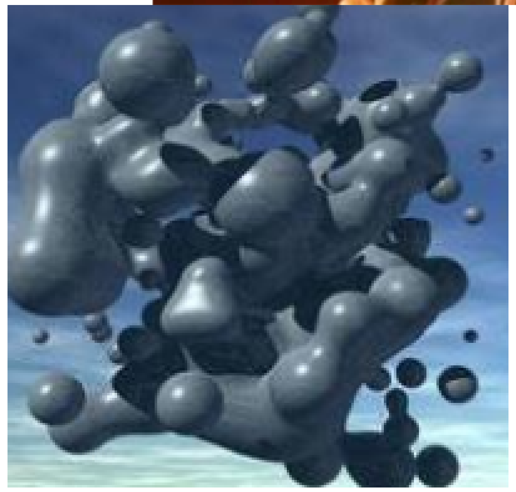
Moving two skeletons toward each other $f(p) = \sum_i f_i(p)$.

[Terminator II 1991]





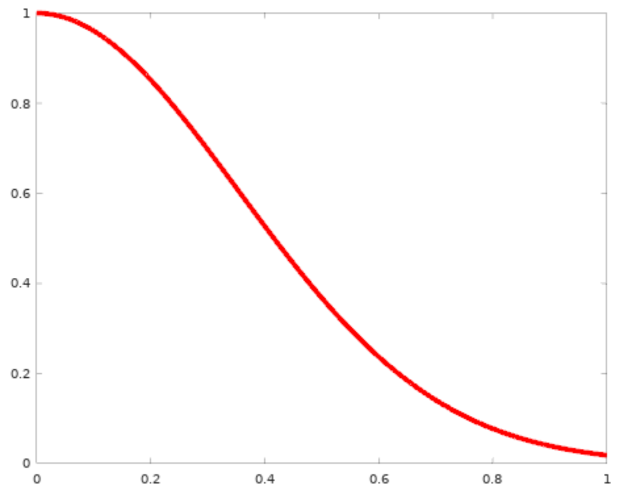




- What is the role of K_i and R_i ?
- What happens when K_i is negative ?
- Model benefits/drawbacks: smoothness ? local control ?

Compact support

Field function with compact support => local control of implicit surface
Distance function $f(d), d > 0$



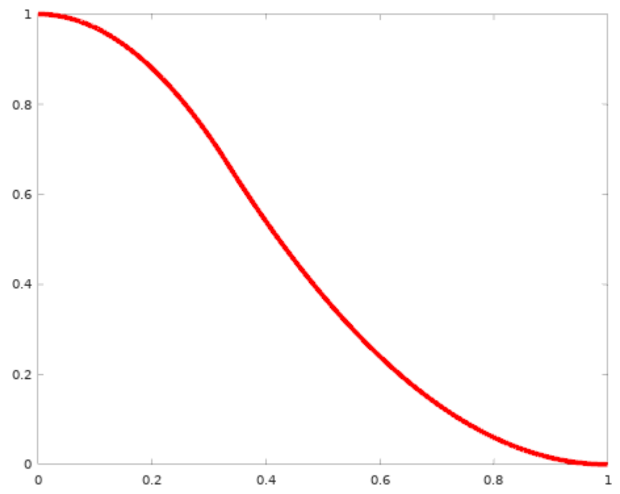
Gaussian

Metaballs

Nishimura 1985

$$f(d) = \begin{cases} 1 - 3\,d^2 & 0 < d \leq 1/3 \\ 3/2\,(1-d)^2 & 1/3 < d \leq 1 \\ 0 & d > 1 \end{cases}$$

C^1 function

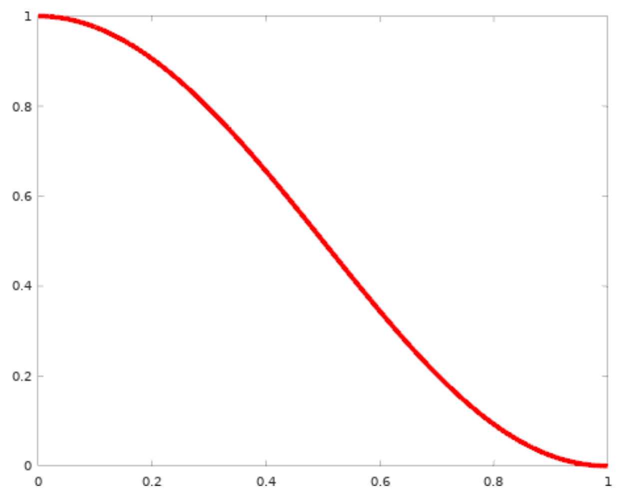


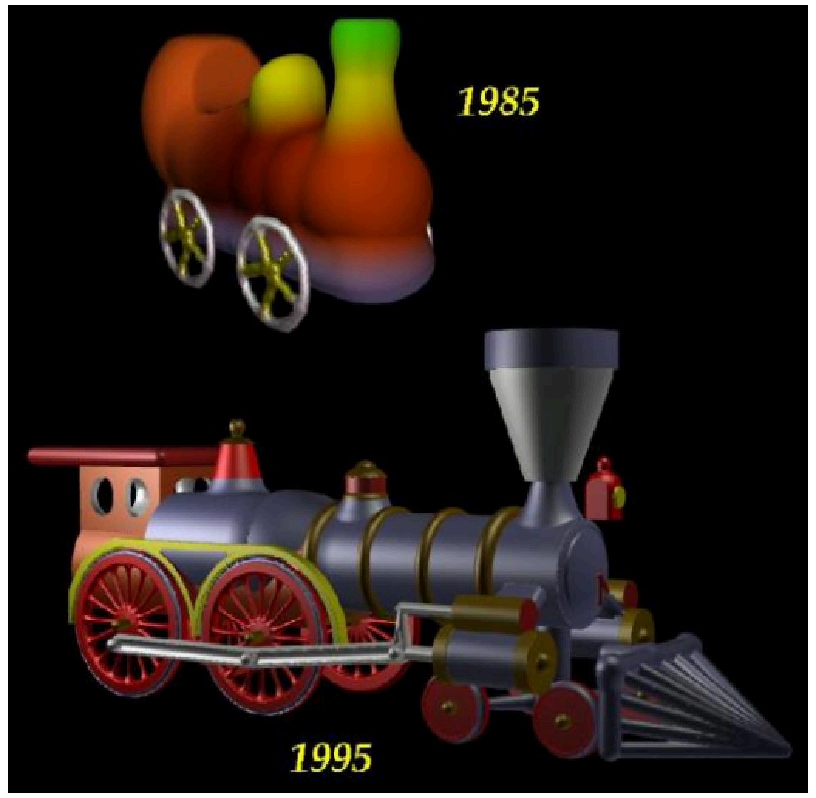
Soft Objects

Wyvill 1986

$$f(d) = -4/9\,d^2 + 17/9\,d^4 - 22/9\,d^2 + 1, 0 < d < 1$$

C^2 function





Transformation

Consider the implicit surface defined as

$f(p) = \exp\left(-\|p\|^2\right)$

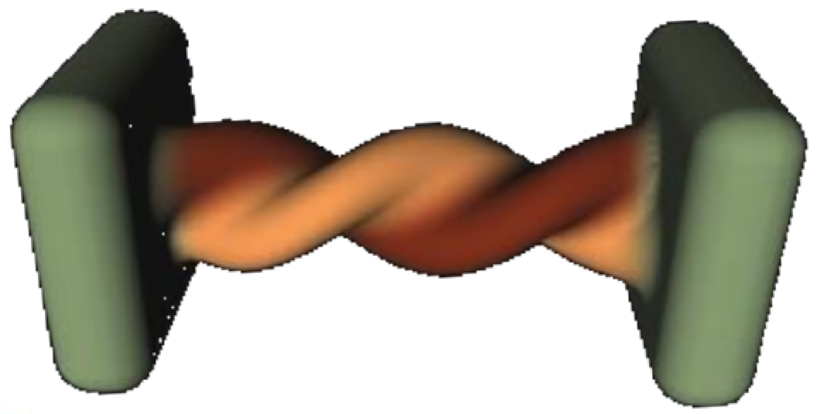
What is the implicit surface corresponding to

- $f(p + (1, 0, 0))$
- $f(2\,p)$

More generally

- $f \circ T(p)$, where T, is a general transformation

Transformation



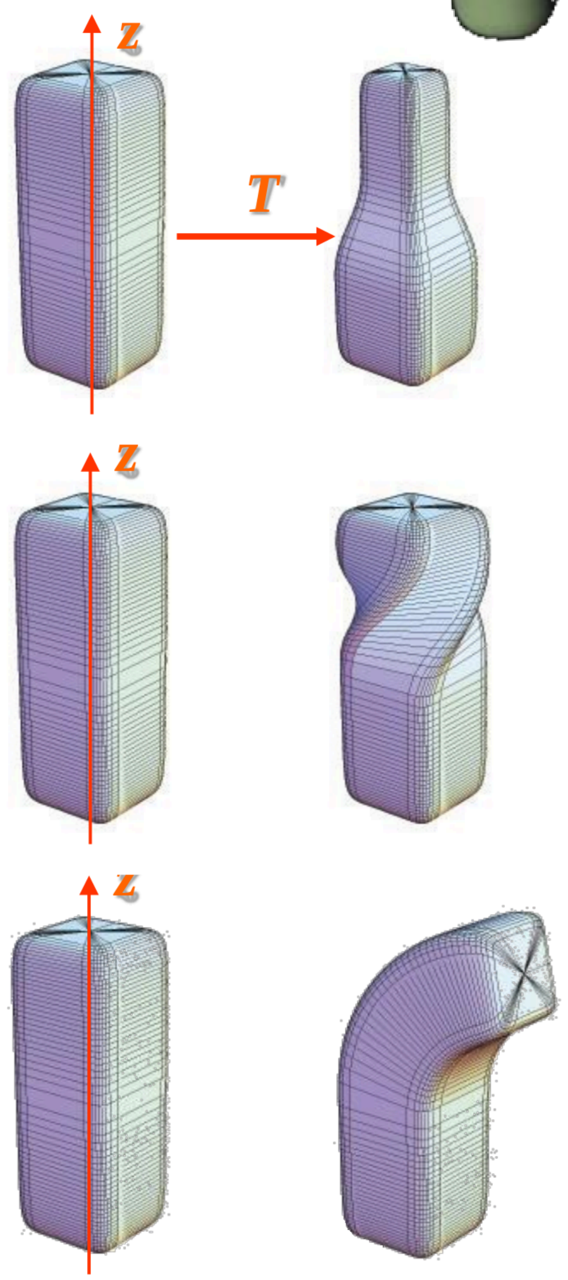
Given a space deformation T on the implicit surface
The associated field is $f_{deformed} = f \circ T^{-1}$.

Common deformations example

$$T = \begin{pmatrix} s(z) & 0 & 0 \\ 0 & s(z) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} \cos(\theta(z)) & \sin(\theta(z)) & 0 \\ -\sin(\theta(z)) & \cos(\theta(z)) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} \cos(\theta(z)) & 0 & -\sin(\theta(z)) \\ 0 & 1 & 0 \\ \sin(\theta(z)) & 0 & \cos(\theta(z)) \end{pmatrix}$$

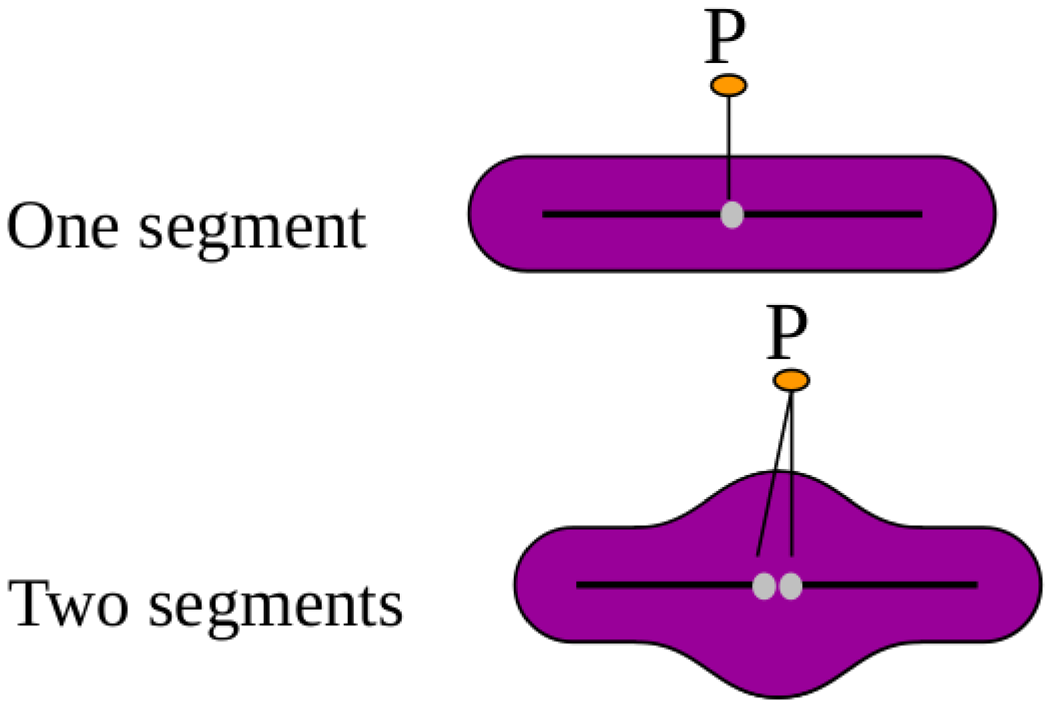


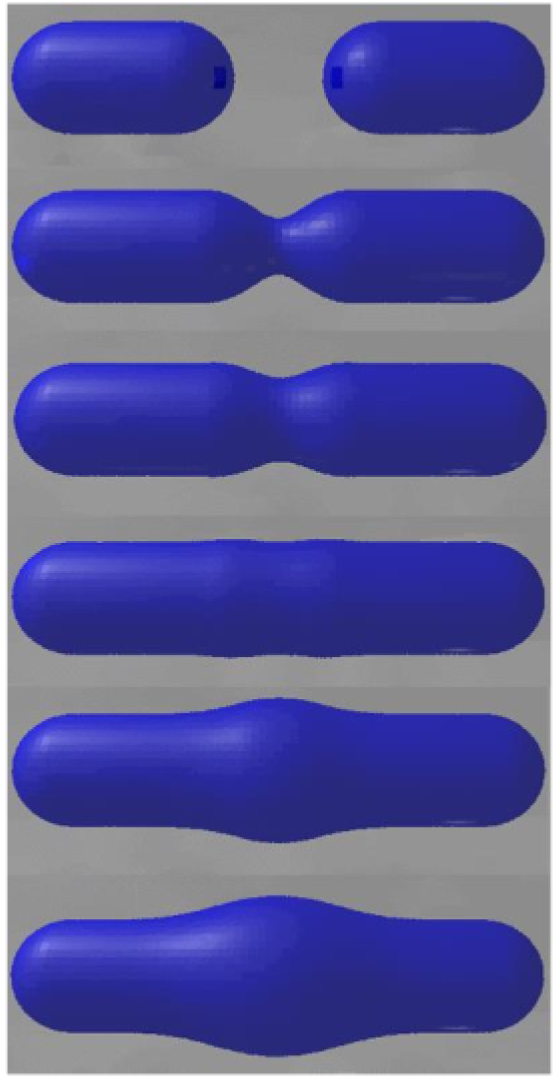
Segment skeleton

- Naive solution: field function based on (shortest cartesian) distance to the segment. Then sum over all segments

  ex. $f(p) = \sum \exp(-d(p,s)^2)$

- Problem: Bulge at junction between two segments.





Convolution surfaces

- Integral of infinitesimal contributions along skeleton

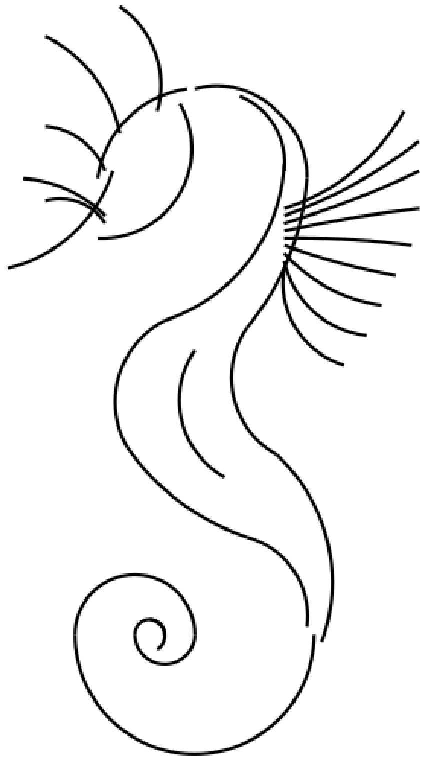
$$f(p) = \int_{q \in skeleton} \kappa(\|p - q\|)\,\mathrm{d}q$$

κ : kernel

Bloomenthal, Shoemake. Convolution Surfaces. ACM SIGGRAPH 1991

- Sum of fields = Field generated by all skeletons

$$f(p) = \sum_i f_i(p) = \int_{q \in C} \sum_i \kappa(\|p - q_i\|)\; dq_i$$



Convolution surfaces: Cauchy Kernel

Cauchy Kernel $\kappa(x) = \dfrac{1}{(1+\alpha^2\, x^2)^2}$

$$f(p) = \int_{q\in squeleton} \frac{1}{(1+\alpha^2\|p-q\|^2)^2}\,\mathrm{d}q,\; q = p_0 + s(p_1 - p_0)\,,\; s\in[0,1]$$

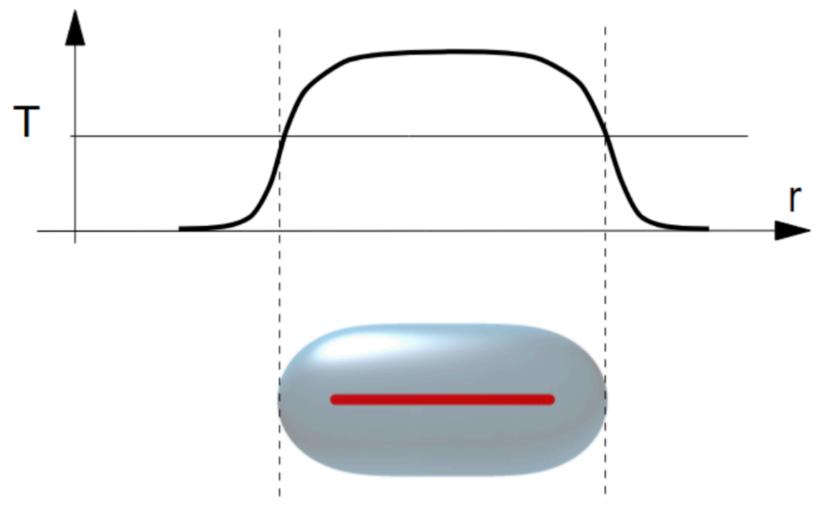
$$f(p) = \frac{1}{2p^2}\left(\frac{h}{p^2+\alpha^2h^2} + \frac{L-h}{\alpha^2(L-h)^2+p^2}\right) + \frac{1}{2\alpha p^3}\left(atan\left(\frac{sh}{p}\right) + atan\left(\frac{s(L-h)}{p}\right)\right)$$

$$L = \|p_1 - p_0\|$$

$$d = \|p - p_0\|$$

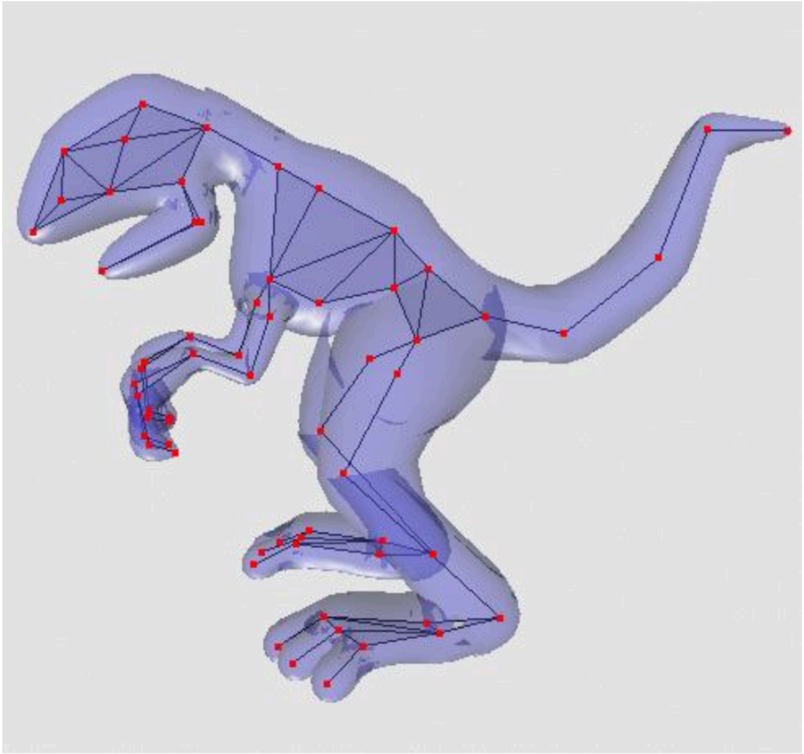
$$h = (p-p_0)\cdot(p_1-p_0)/L$$

$$p = 1+\alpha^2(d^2-h^2)$$



Generalization to triangular skeleton

$$f(p) = \iint_{q \in squeleton} \kappa(\|p-q\|)\mathrm{d}q$$



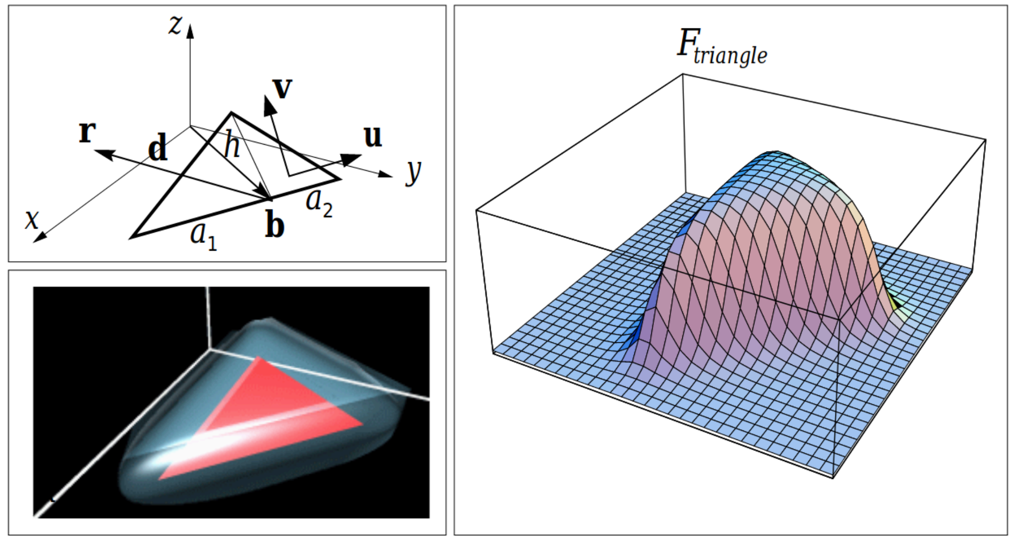
Explicit formula for Cauchy Kernel

J. McCormack, A. Sherstyuk. Creating and Rendering Convolution Surfaces. CGF 98.

$$F_{triangle}(\mathbf{r}) =$$
$$= \frac{1}{2qs}\left(\frac{n}{A}(\mathrm{atan}[\frac{s(vh+a_1(a_1+u))}{A}] + \mathrm{atan}[\frac{s(gh+a_1u)}{-A}]) + \right.$$
$$+\frac{m}{B}(\mathrm{atan}[\frac{s(vh+a_2(a_2-u))}{-B}] + \mathrm{atan}[\frac{s(gh-a_2u)}{B}]) +$$
$$\left. +\frac{v}{C}(\mathrm{atan}[\frac{s(a_1+u)}{C}] + \mathrm{atan}[\frac{s(a_2-u)}{C}])\right),$$

where

$$A^2 = a_1^2 w + h^2(q+s^2u^2) - 2hs^2a_1ug,$$
$$B^2 = a_2^2 w + h^2(q+s^2u^2) + 2hs^2a_2ug,$$
$$C^2 = 1 + s^2(d^2 - u^2),$$
$$g = v - h,$$
$$q = 1 + s^2(d^2 - u^2 - v^2),$$
$$w = c^2 - 2hs^2v + h^2s^2,$$
$$m = a_2 g + uh,$$
$$n = uh - a_1 g$$



Blending Operators

- $\max(f_1(p), f_2(p)), \min(f_1(p), f_2(p)), f_1(p) + f_2(p)$ are not the only possible operators

- Ricci operators (*Clean Union*)

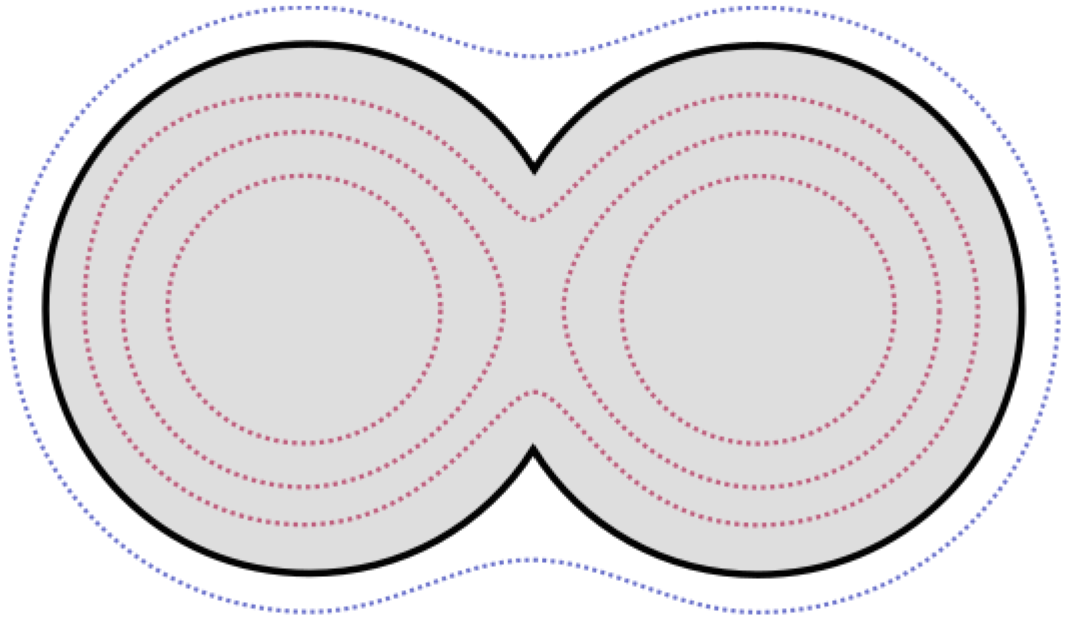
A. Ricci. A constructive geometry for computer graphics. Computer Journal, 1973.

$f(p) = (f_1^n(p) + f_2^n(p))^{1/n}$

- Pasko operators

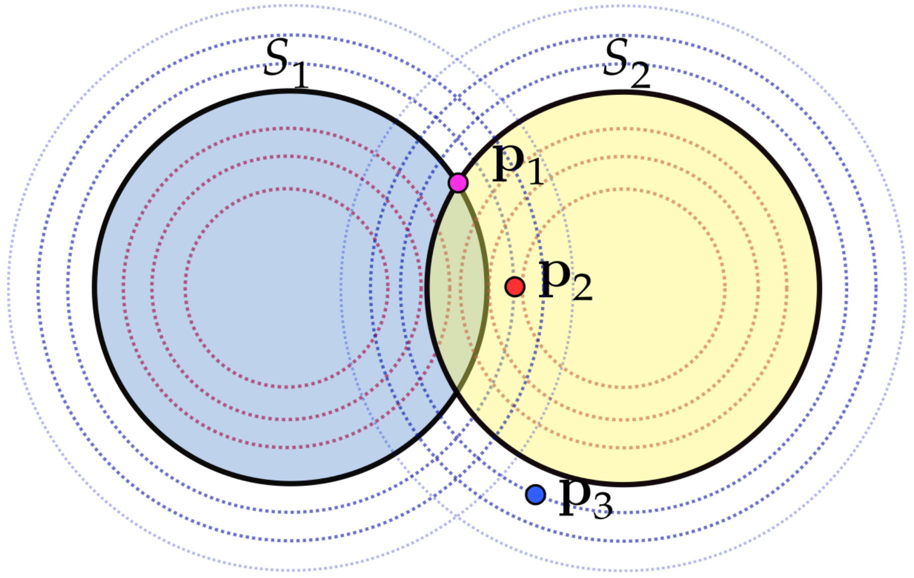
A. Pasko et al. Function Representation in Geometric Modeling. TVC 95

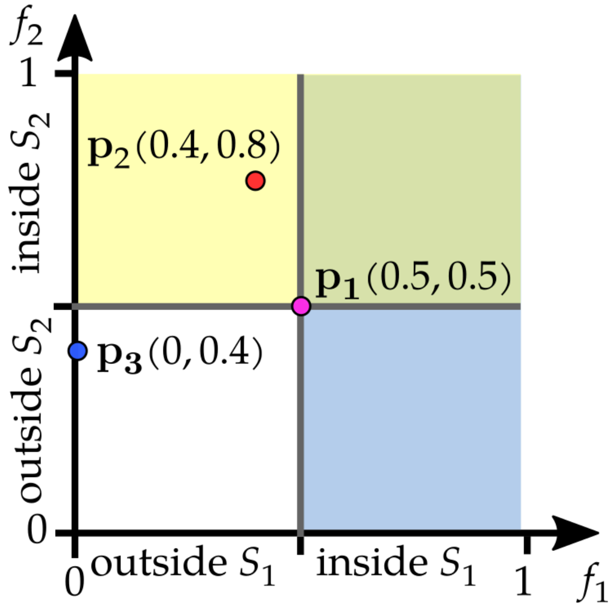
$$f(p) = f_1(p) + f_2(p) - \sqrt{f_1^2(p) + f_2^2(p)} + \frac{a_0}{1 + \left(\frac{f_1}{a_1}\right)^2 + \left(\frac{f_2}{a_2}\right)^2}$$

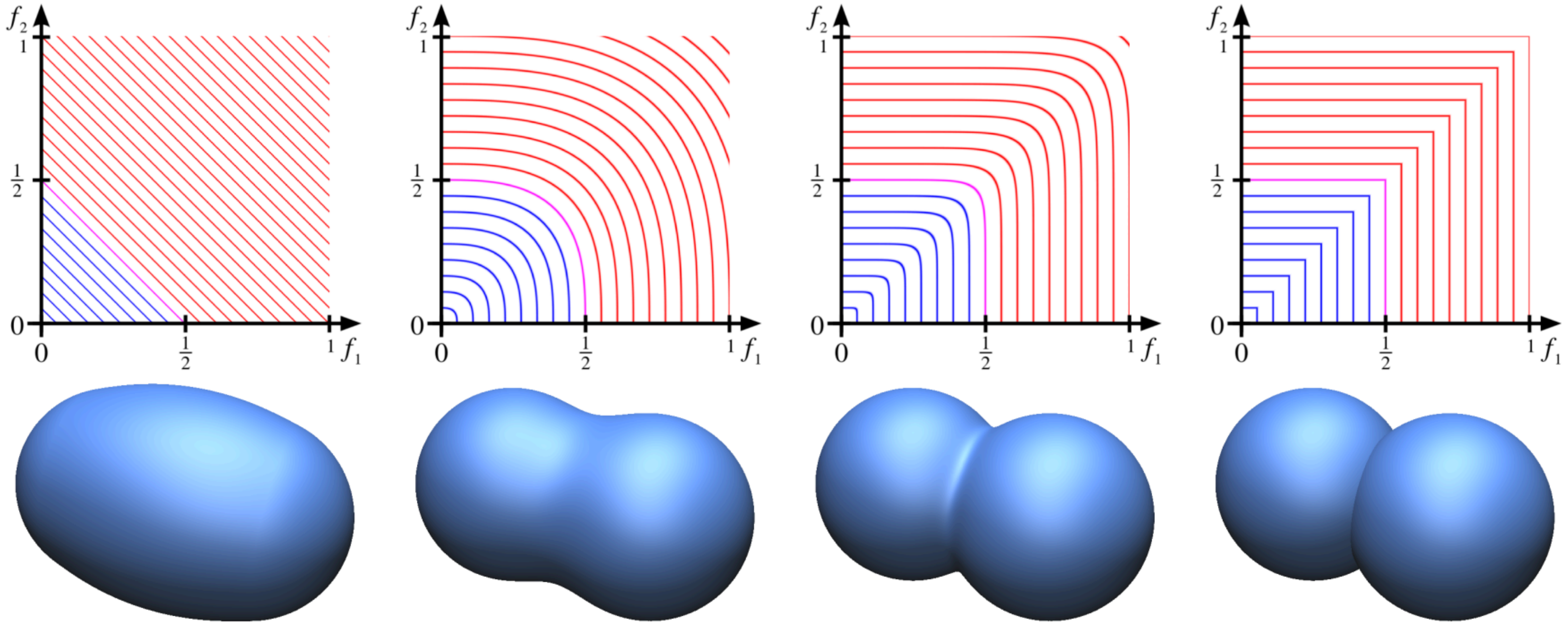


Graphical interpretation of blending operators

Representation in the space $f_1 - f_2$







(a) Sum operator **(b)** Ricci-3 operator **(c)** Ricci-10 operator **(d)** Union operator

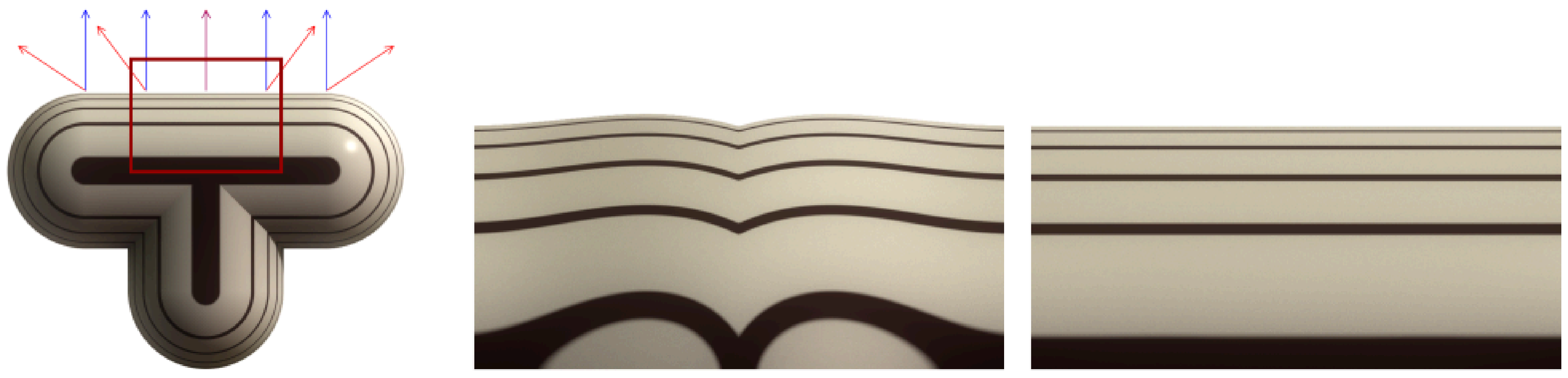
images from Valentin Roussellet

Gradient Blending

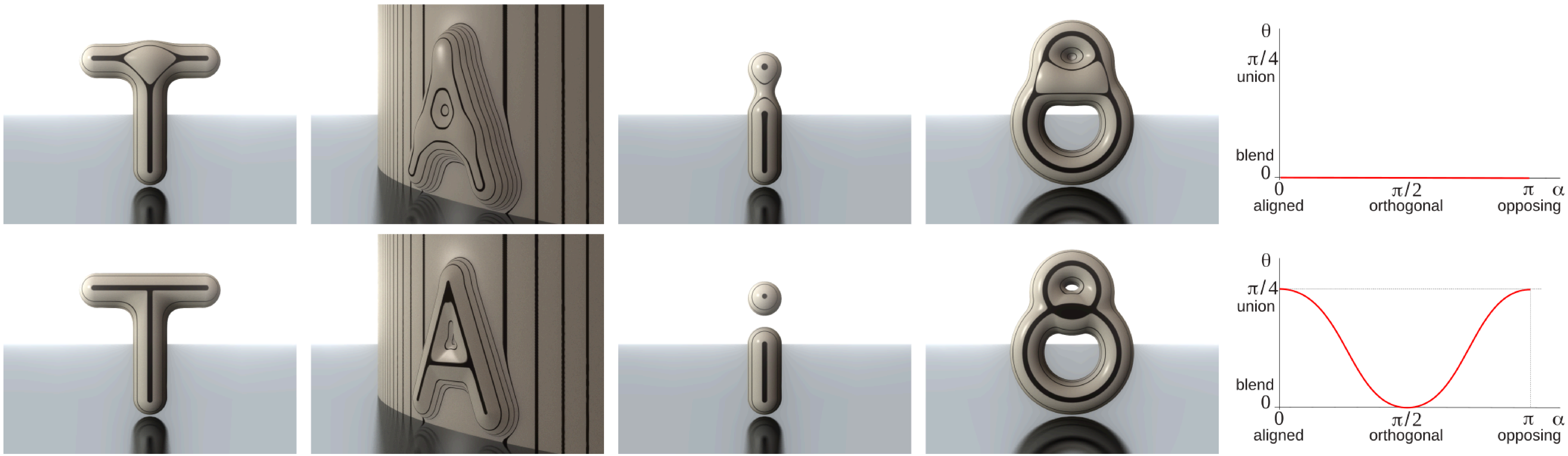
Taking into account gradients of fields

$f = \mathcal{O}(f_1, f_2, \nabla f_1, \nabla f_2)$

O. Gourmel et al. A Gradient-Based Implicit Blend. TOG 201:



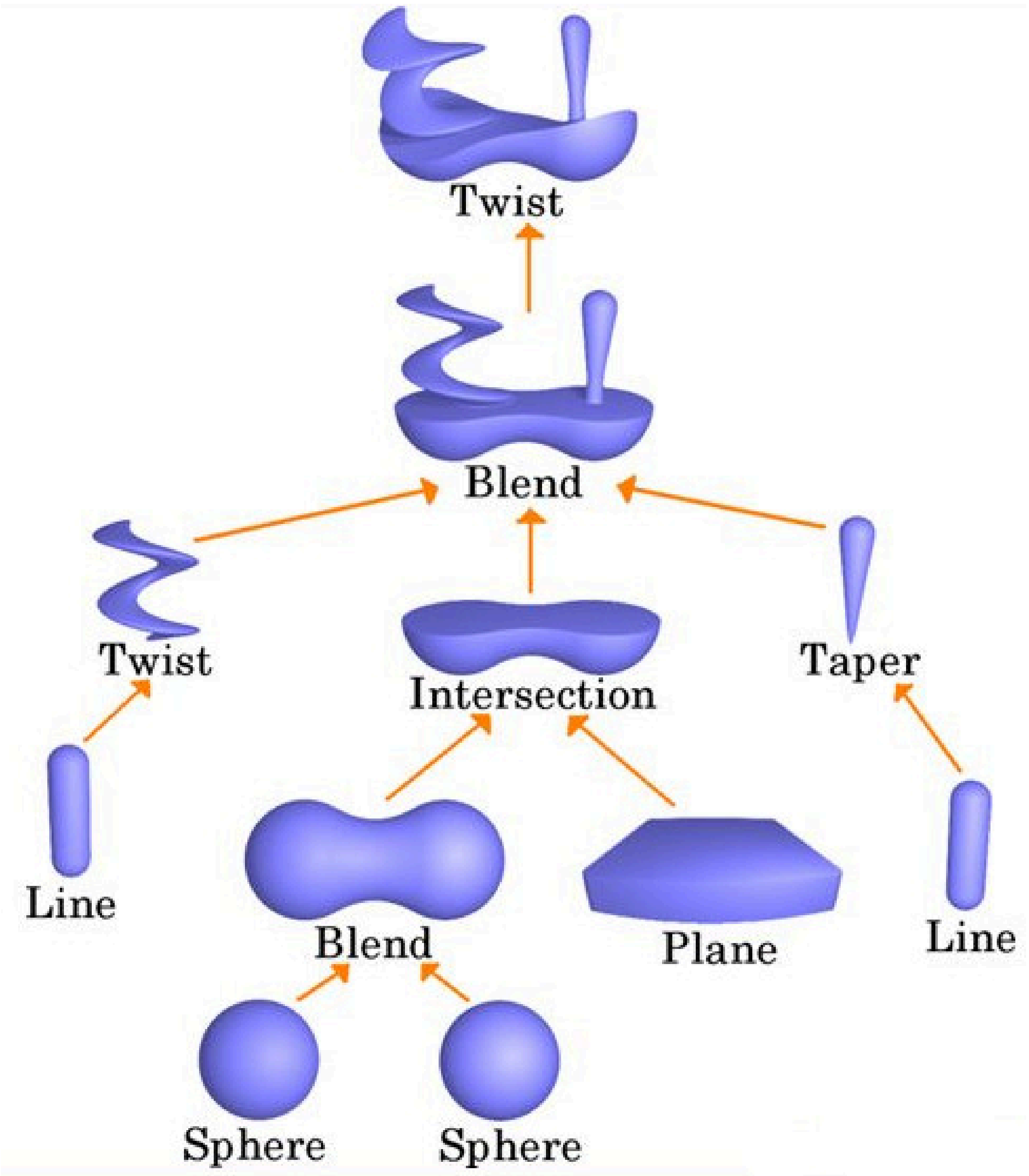
(a) (b) (c)



Blob Tree

Modeling a CSG construction tree

- Terminal leaves are skeleton
- *Binary/N-ary edges* are operators
- *Unary edges* are space deformations



Erwin de Groot and Brian Wyvill